Signalling in Auctions for Risk-Averse and Loss-Averse Bidders

Olivier Bos*, Francisco Gomez-Martinez**, Sander Onderstal***

25 March 2022

Abstract:
We study risk-averse and loss-averse equilibrium bidding in first-price and second-price sealed-bid auctions where bidders have signalling concerns, i.e., they care about how the auction outcome is interpreted by an outside observer. We find that when the winner's identity and her payment are revealed to the outside observer, risk aversion and loss aversion yield less aggressive bidding behaviour in the second-price sealed-bid auction than in the risk-neutral case. Our analysis explains various revenue ranking reversals relative to the risk-neutral equilibrium observed in a recent experiment by Bos *et al.* (2021).

Keywords: Auctions; Signalling; Risk aversion; Loss aversion
JEL classification: D44; D81; D82

Acknowledgments: We are grateful to Tom Truyts for helpful comments. We thank the ANR SIGNAL (ANR-19-CE26-0009) for its financial support.

* Université Paris-Saclay, ENS Paris-Saclay, Centre d'Economie de l'ENS Paris-Saclay and ZEW-Leibniz Centre for European Economic Research, olivier.bos@ens-paris-saclay.fr.

** Department of Economics, BI Norwegian Business School, francisco.g.martinez@bi.no.

*** University of Amsterdam and Tinbergen Institute, onderstal@uva.nl.

1. Introduction

In the pastdw two decades, auctions in which bidders care about how an outside observer interprets the auction outcome have been studied extensively. Applications include art selling, charitable fundraising, takeover bidding, competing firms issuing equity, and procurement. In such settings, bidders are expected to condition their bidding strategies on the signalling opportunities. Various disclosure policies, including revealing the winner's identity, the (winning) bids, and the (winner) payments, have been shown to affect equilibrium bidding in auctions. Goeree (2003), Liu (2012), Molnar and Virag (2008), Giovannoni and Makris (2014), Dworzcak (2020), and Fonseca et al. (2020) examine how revealing the winner's identity and the bids to an outside observer affects equilibrium bidding. Bos and Pollrich (2022), Bos et al. (2021), and Bos and Truyts (2021) consider the effect of revealing the winner's identity and the payments.

The received literature has mainly focused on the risk-neutral case, ignoring risk-averse and loss-averse bidding strategies. This is surprising as both risk aversion and loss aversion have been shown to explain various observations in auctions better than risk neutrality, both in the lab and in the field. Cox *et al.* (1985, 1988) and Isaac and James (2000) argue that risk aversion explain bidding behavior in the laboratory. Camerer (1995) and Lange and Ratan (2010) explain bidding behavior in the lab using loss aversion. For field evidence, see Athey and Levin (2001), Lu and Perrigne (2008), Campo et al. (2011), and Kong (2020) for risk-averse bidders, and Banerji and Gupta (2014) for loss-averse bidders.

A prominent example is the experimental observation in the symmetric-independent-private-values paradigm that the first-price sealed-bid auction (FPSB) yields higher revenue on average than the second-price sealed-bid auction (SPSB), violating the famous revenue-equivalence result for the case of risk-neutral bidders (Vickrey, 1961; Myerson, 1981). The recent experiment by Bos, Gomez-Martinez, Onderstal and Truyts (2021), hereafter BGOT, highlights the need to understand the effect of risk-averse and loss-averse preferences in auction settings with signalling opportunities. In such a setting, BGOT observe that submitted bids deviate notably from the risk-neutral Bayesian perfect Nash equilibrium. More specifically, (i) bidders overbid/underbid systematically in FPSB/SPSB;[1] (ii) FPSB yields

[1] In an experiment studying various disclosure policies in FPSB, Fonseca *et al.* (2020) also observe consistent overbidding relative to the risk-neutral equilibrium.

higher revenue than SPSB; (iii) revenue in SPSB is higher when only the winner is revealed than when the winner and her payment are revealed.

In the current paper, we study theoretically whether risk-averse or loss-averse equilibrium bidding can explain the anomalies observed in BGOT's experiment. We obtain the following results. In FPSB, risk aversion leads to more aggressive equilibrium bidding compared to the risk-neutral case. Intuitively, like in the standard case where bidders' utilities are not affected by how an outside observer interprets their bids, risk-averse bidders mitigate the risk of losing by submitting a higher bid than risk-neutral bidders. Moreover, when both the winner identity and her payment are revealed, risk-averse bidders have an additional incentive to bid aggressively because the winner gets a certain payoff from signalling opportunities while a loser's payoff is random as the payoffs only depend on the winner's bid. In SPSB, risk aversion and loss aversion do not affect the equilibrium in weakly dominant strategies. When both the winner identity and her payment are revealed, risk-averse and loss-averse bidders bid less aggressively than risk-neutral bidders. The reason is that winning in the SPSB is relatively unattractive because payoffs from signalling opportunities are random in that they depend on the second highest bid. The observation that risk aversion and loss aversion depress bids in SPSB is in contrast to the standard independent-private-values model without signalling opportunities, where risk attitude nor loss attitude affects the weakly dominant strategy of bidding value.[2] Our findings offer a potential explanation for the above experimental results in BGOT.

The remainder of this paper is organized as follows. In Section 2, we describe our model. Sections 3 and 4 contain our findings for risk-averse and loss-averse bidders respectively. Section 5 is a short conclusion.

2. The model

We consider a setting with $n \geq 2$ (female) bidders, indexed $i = 1, \ldots, n$, who compete for a single object allocated through an auction. Each bidder i is privately informed about her value v_i (her 'type'). The v_i's are i.i.d. drawn from a smooth distribution function F on $[0, \bar{v}]$, $\bar{v} > 0$, which admits a continuous density function $f \equiv F'$. The auction outcome is partly revealed to an outside observer (male). We study settings where the outside observer either observes the

[2] See Riley and Samuelson (1981) and Maskin and Riley (1984) for an analysis of equilibrium bidding for risk-averse bidders in the independent-private-values model.

winner's identity and her payment, or only the winner's identity. Our model reflects economic situations in which bidders care about how they are perceived by others. For instance, the fact that a firm wins or loses an auction could be interpreted by market analysts as a signal of the firm's management quality. Another example is when outsiders consider a high bid in a charity auction as informative about how much the bidder cares about the charity. Our assumption that in some settings the winner's payment is communicated to outsiders is not without application either: A directive on public procurement in the European Union stipulates that payments by the bidders should be publicly revealed in a contract award notice.[3]

We assume that bidders not only care about winning the auction, but also about the inference of the outside observer about her type. Following BGOT, we assume that a bidder's payoff is increased by $\gamma\tilde{v}$ ($\gamma > 0$), where $\tilde{v}$ denotes the outside observer's best estimate of the bidder's value based on what he learns about the auction outcome that is revealed. The resulting payoff for bidder i is given by

$$\pi_i = \begin{cases} v_i - p + \gamma\tilde{v} & \text{if } w = i \\ \gamma\tilde{v} & \text{if } w \neq i \end{cases},$$

where w denotes the auction winner and p the winner's payment. The parameters is BGOT's experiment are $\gamma = 1/2$, $n = 3$, and $F = U[0,1]$. BGOT compare FPSB and SPSB in two settings. In the first setting, the winner's identity is disclosed to the outside observer. This setting is labelled FPW and SPW for FPSB and SPSB respectively. In the other setting, labelled FPWP and SPWP respectively, both the winner identity and her payment are revealed to the outside observer.

All bidders evaluate their payoffs according to utility function $u: \mathbb{R} \to \mathbb{R}$. In the remainder of the paper, we compare risk-neutral bidders, where $u(x) = x$ for all $x \in \mathbb{R}$, to risk-averse and loss-averse bidders. As all bidders share the same beliefs about the other bidders' values, we assume them to follow a symmetric bidding strategy. B denotes the strictly increasing risk-neutral bidding function and A denotes the strictly increasing risk-averse or loss-averse bidding function, with $B, A: [0, \bar{v}] \to \mathbb{R}_+$.[4]

[3] See Annex V part D of Directive 2014/24/EU of the European Parliament and of the Council of 26 February 2014 on public procurement.
[4] To avoid messy notation, we do not use labels for auction, the case, the level of risk aversion, or the level of loss aversion, unless indicated otherwise.

The risk-neutral analysis follows straightforwardly from Giovannoni and Makris (2014) and Bos and Truyts (2021). We restrict our attention to the perfect Bayesian Nash equilibria that survive Banks and Sobel's (1987) D1 criterion (referred to as "equilibrium" in the remainder of this paper). Table 1 presents the risk-neutral equilibrium bids and expected revenue for the experimental parameters as well as the average revenue observed in BGOT's experiment. Three observations stand out from the experimental findings: (i) overbidding/underbidding in FPSB/SPSB relative to the risk-neutral equilibrium predictions; (ii) comparing FPW to SPW, and FPWP to SPWP, FPSB yields higher revenue than SPSB, in contrast to the risk-neutral equilibrium predictions; (iii) revenue in SPSB is higher in SPW than in SPWP, in contrast to the risk-neutral equilibrium predictions. We investigate risk-averse and loss-averse bidding in an attempt to understand these three anomalies.

Table 1: Equilibrium bids and revenue for risk-neutral bidders for $\gamma = 1/2, n = 3, F = U[0,1]$

Case	Auction	Information to the outside observer	Equilibrium bids	Expected revenue	Average revenue in BGOT's experiment
FPW	FPSB	The winner	$B(v) \approx \frac{2}{3}v + 0.22$	$R \approx 0.72$	0.7319
FPWP	FPSB	The winner and her payment	$B(v) = v$	$R = 0.75$	0.8039
SPW	SPSB	The winner	$B(v) \approx v + 0.22$	$R \approx 0.72$	0.6830
SPWP	SPSB	The winner and her payment	$B(v) = \frac{v}{2} + 5/8$	$R = 0.875$	0.6475

3. Risk-averse bidders

We first explore risk aversion as a potential driver for more aggressive bidding in FPSB with signalling incentives and underbidding behaviour compared to predictions for SPSB and the consequences for the revenue ranking reversal. In order to do so, we derive the equilibrium bidding curves for risk-averse bidders and compare them with the equilibrium bidding curves for risk-neutral bidders (Giovannoni and Makris, 2014; Bos and Truyts, 2021). In the case of risk-averse preferences, we assume that u is twice differentiable with $u' > 0$ and $u'' < 0$. Following Matthews (1983), to simplify the analysis of SPSB when both winner's identity and her payment are disclosed, we consider constant absolute risk aversion preferences (CARA), i.e., $u(x) = -\exp(-\alpha x)$ with $\alpha > 0$.

The remainder of this section is structured as follows. In Section 3.1, we establish the role of risk aversion in FPW and SPW. In Section 3.2, we establish that, relative to the risk-neutral case, risk aversion shifts equilibrium bidding curves upwards in FPWP. In Section 3.3, we derive equilibrium bidding curves with risk aversion in SPWP and show that risk aversion depresses equilibrium bidding under CARA preferences. Finally, in Section 3.4, we compare expected revenues in FPWP and SPWP assuming CARA preferences and show that for sufficiently risk averse bidders, FPSB yields higher expected revenue than SPSB.

3.1 FPW and SPW

For the setting where only the identity of the winning bidder is revealed, let k_w and k_l denote the outside observer's equilibrium value estimates for the winner and the losing bidders respectively. Because these quantities do not depend on the bids submitted on the equilibrium path, the effect of risk aversion on equilibrium bidding follows from standard reasoning. In particular, in SPSB, bidding value plus $\gamma(k_w - k_l)$ is a weakly dominant strategy, regardless of risk attitude. In FPSB, the risk-averse equilibrium is readily derived by inflating all values by $\gamma(k_w - k_l)$. As a result, the standard result that FPSB generates a higher revenue than SPSB in the case of risk-averse bidders generalizes to our setting if only the winner's identity is revealed. BGOT's experimental observations are in line with this result (see Table 1), although the difference between the auctions is not statistically significant.

Proposition 1. *Suppose only the winner's identity is revealed to the outside observer. Then, (i) risk-averse bidders bid more aggressively in equilibrium than risk-neutral bidders in FPSB, (ii) equilibrium bidding in SPSB is not affected by risk aversion, and (iii) in equilibrium, expected revenue is greater in FPSB than in SPSB.*

3.2 FPWP

We start by deriving a symmetric equilibrium for FPWP. If the other bidders stick to the equilibrium bidding curve, a bidder with a type v, pretending to be a type $w \in [0, \bar{v}]$, faces the following problem:

$$\max_w U(v,w) = F^{(1)}(w)u(v - A(w) + \gamma w) + \int_w^{\bar{v}} u(\gamma \tilde{V}(x))\, dF^{(1)}(x).$$

where $F^{(1)}$ denotes the distribution of the highest-order statistic of $n-1$ i.i.d. draws from F. The first term on the RHS refers to the case in which the bidder wins and then the outside observer induces that the bidder's value equals w. The second term is the bidder's payoff when losing the auction, where $\tilde{V}(x)$ denotes the outside observer's optimal value estimate for losing bidders if the winning bidder's value equals x. The equilibrium FOC is given by

$$\left.\frac{\partial U(v,w)}{\partial w}\right|_{w=v} = f^{(1)}(v)u(v-A(v)+\gamma v) - F^{(1)}(v)u'(v-A(v)+\gamma v)(A'(v)-\gamma) - u(\gamma\tilde{V}(v))f^{(1)}(v) = 0$$

where $f^{(1)}$ is the density function corresponding to $F^{(1)}$. It implies that

$$\begin{aligned} A'(v) &= \frac{f^{(1)}(v)}{F^{(1)}(v)}\frac{u\big(v(1+\gamma)-A(v)\big)-u\left(\gamma\tilde{V}(v)\right)}{u'\big(v(1+\gamma)-A(v)\big)}+\gamma \\ &> \frac{f^{(1)}(v)}{F^{(1)}(v)}\left(v(1+\gamma)-A(v)-\gamma\tilde{V}(v)\right)+\gamma \qquad (1) \end{aligned}$$

To establish (1), notice that $\frac{u(v)-u(z)}{u'(v)} > v-z$ for all $v>z$ and that in equilibrium the winning payoff is always higher than the losing payoff. Then, using the same technical arguments as Krishna (2009, Chapter 4, page 39), we get

$$A(v) > B(v)$$

for all $v>0$. We can then conclude with the following proposition:

Proposition 2. *If the winner identity and her payment are revealed to the outside observer, risk-averse bidders bid more aggressively than risk-neutral bidders in FPSB.*

Therefore, risk aversion may explain the overbidding observed for the FPWP in BGOT (see Table 1). As we will show in Section 3.4, the overbidding implied by risk aversion can be sufficiently strong to reverse the ranking of expected revenues between FPSB and SPSB, consistent with BGOT's data.

3.3 SPWP

The analysis for SPWP proceeds analogously to that for FPWP. Let $V(x)$ and $L(x)$ denote the outside observer's estimate of the winner's value and the loser's value respectively, conditional on the second-highest value being x. Recall that the identity of the second highest bidder is unknown to the outside observer. A type v loser is the second highest bidder with probability $H(v) = (n-1)F^{n-2}(v)(1-F(v))$ and another bidder with probability $K(v) = (n-1)F^{n-2}(v) - (n-2)F^{n-1}(v)$. We denote h and k the density functions associated with H and K respectively. Therefore, a bidder with a type v, pretending to be a type w, faces the following problem:

$$\max_{w} U(v,w) = \int_0^w u(v - A(x) + \gamma V(x))\, dF^{(1)}(x) + H(w)u\big(\gamma L(w)\big) + \int_w^{\bar{v}} u\big(\gamma L(x)\big)\, dK(x),$$

with $V(x) = \frac{\int_x^{\bar{v}} y dF(y)}{1-F(x)}$ and $L(w) = \frac{w}{n-1} + \frac{n-2}{n-1}\frac{\int_0^w x dF(x)}{F(w)}$. The equilibrium FOC is given by

$$\left.\frac{\partial U(v,w)}{\partial w}\right|_{w=v} = f^{(1)}(v)u(v - A(v) + \gamma V(v)) + h(v)u\big(\gamma L(v)\big) + H(v)u'\big(\gamma L(v)\big)\gamma L'(v) - u(\gamma L(v))k(v) = 0.$$

Given the CARA preferences $u(x) = -\exp(-\alpha x)$ for $\alpha > 0$, and using the parameters from the experiment ($\gamma = 1/2,\ n = 3,\ F = U[0,1]$), we have $V(v) = \frac{1+v}{2}, L(v) = \frac{3}{4}v, F^{(1)}(v) = v^2, H(v) = 2v(1-v), K(v) = v(2-v)$ and $F^{(1)}(v) = v^2$. Then, the FOC becomes

$$-\exp\left(\alpha A(v) - \alpha\frac{5v+1}{4}\right)2v - 2(1-2v)\exp\left(-\frac{3\alpha}{8}v\right) + 2v(1-v)\exp\left(-\frac{3\alpha}{8}v\right)\frac{3\alpha}{8} + 2(1-v)\exp\left(-\frac{3\alpha}{8}v\right) = 0.$$

Therefore,

$$A(v) = \frac{7}{8}v + \frac{1}{4} + \frac{1}{\alpha}\ln\left(1 + \frac{3\alpha}{8}(1-v)\right) \qquad (2)$$

We prove $A(v) < B(v)$ by contradiction. Recall from Table 1 that $B(v) = v/2 + 5/8$. Then,

$$A(v) \geq B(v) \Leftrightarrow$$

$$\frac{7}{8}v + \frac{1}{4} + \frac{1}{\alpha}\ln\left(1 + \frac{3\alpha}{8}(1-v)\right) \geq v/2 + 5/8 \Leftrightarrow$$

$$\ln\left(1 + \frac{3\alpha}{8}(1-v)\right) \geq \frac{3\alpha}{8}(1-v) \Leftrightarrow$$

$$1 + \frac{3\alpha}{8}(1-v) \geq \exp\left(\frac{3\alpha}{8}(1-v)\right),$$

which is in contradiction with $\exp(x) > x + 1$ for all $x > 0$. Finally, note that $A(0) < B(0)$ for all $\alpha > 0$. This result is summed up in the following proposition:

Proposition 3. *Assume $\gamma = 1/2$, $n = 3$, $F = U[0,1]$ and consider CARA preferences. If the winner identity and her payment are revealed to the outside observer, risk averse bidders bid less aggressively than risk-neutral bidders in SPSB.*

For SPWP, equilibrium revenue for CARA risk preferences is lower than in the risk-neutral case. Intuitively, in SPWP, winning is relatively unattractive because the winner faces a random outside observer's estimate as it depends on the second highest bid. Indeed, in the experimental data generated in BGOT, revenue in SPWP is much lower than in the risk-neutral equilibrium (see Table 1). This is a surprising result in view of the standard case without an outside observer in which bidding behavior is not affected by risk aversion in SPSB.

3.4 Revenue comparison

We now compare FPSB and SPSB in terms of expected equilibrium revenue under BGOT's experimental parameters assuming CARA preferences. Let R_α^T denote the revenue for case T, i.e. FPW, FPWP, SPW, or SPWP, where bidders' degree of risk aversion equals $\alpha \geq 0$. Let A_∞ the bidding strategy for SPWP in the case of infinitely risk averse bidders, i.e., $A_\infty(v) = \lim_{\alpha \to +\infty} A(v)$ for all $v \in [0,1]$. Equation (2) implies

$$A_\infty(v) = \frac{7}{8}v + \frac{1}{4}$$

because $\lim_{\alpha \to +\infty} \frac{1}{\alpha} \ln\left(1 + \frac{3\alpha}{8}(1 - v)\right) = 0$. It follows that the associated expected revenue, R_{∞}^{SPWP}, is equal to $\frac{11}{16}$. Compared to the risk-neutral case, risk aversion affects equilibrium bidding strategies positively in FPSB and negatively in SPSB, and therefore can lead to a lower revenue in the later:

$$R_{\infty}^{SPWP} = \frac{11}{16} = 0.6875 < R_0^{FPWP} = \frac{3}{4} < R_{\alpha>0}^{FPWP}$$

where R_0^{FPWP} denotes expected revenue in the risk-neutral equilibrium of FPWP (see Table 1). In other words, a sufficiently high level of risk aversion can explain the revenue ranking swap observed in the data relative to the risk-neutral case.

SPWP with infinitely risk-averse bidders also yields lower revenue than SPW:

$$R_{\infty}^{SPWP} = \frac{11}{16} = 0.6875 < R_0^{SPW} = R_{\alpha>0}^{SPW} = \frac{1}{2} + (k_w - k_l) \approx 0.72.$$

While with risk-neutral bidders $R_0^{SPW} < R_0^{SPWP}$ and R_0^{FPWP}<R_0^{SPWP}, our computations lead to the opposite rankings for sufficiently risk-averse bidders:

Proposition 4. *Assume $\gamma = 1/2$, $n = 3$, $F = U[0,1]$. For sufficiently risk-averse bidders with CARA preferences, the revenue ranking of SPWP with SPW and FPWP swap compared to the risk-neutral case.*

This result is qualitatively consistent with the data collected in BGOT (see Table 1). Therefore, risk aversion is again a potential candidate to explain the discrepancy between the data and the theoretical predictions. In SPW, equilibrium bids are unaffected: bidders still have a weakly dominant strategy to bid value plus the (deterministic) payoff from the outside observer providing a higher estimate for the winner than for the losers. As said, for SPWP, CARA bidders submit lower bids than risk-neutral bidders. We find that for sufficiently risk averse bidders, SPWP yields lower expected revenue that SPW.

In contrast to SPWP, risk aversion produces an increase in equilibrium bids in FPWP compared to the risk-neutral case. The intuition is straightforward. First of all, like in the standard case, a bidder mitigates the risk of losing the auction by submitting a high bid. Second, in our setting, a risk averse bidder has an additional incentive to win because the winner obtains a sure payoff from the outside observer's estimate, which is deterministic for the winning bid, while a loser faces a stochastic estimate that depends on the winner's bid. As a consequence, for sufficiently risk averse bidders, the revenue rankings between SWP and SPWP on the one hand and between FPWP and SPWP on the other reverse, in line with BGOT's experimental findings (see Table 1).

4. Loss-averse bidders

Now, we explore loss aversion as a potential driver of the deviations from risk-neutral equilibrium predictions observed in BGOT's experiment. In order to do so, we derive the equilibrium bidding curves for loss-averse bidders and compare them with the equilibrium bidding curves for risk-neutral bidders.

We assume the standard loss aversion function (Kahneman and Tversky, 1979, 1992):

$$u(x) = \begin{cases} x^{\theta} \ if \ x \geq \ 0 \\ -\beta(-x)^{\theta} \ otherwise \end{cases}$$

with $\theta > 0$ and $\beta \geq 1$. Risk-neutral utility corresponds to $\theta = \beta = 1$.

The remainder of this section is organized as follows. In Section 4.1, we will observe that loss aversion does not affect bids in FWP and SPW. In Section 4.2, we show that loss aversion depresses equilibrium bidding in SPWP. In Section 4.3, we compare expected revenues between FPSB and SPSB in the case of loss-averse bidders.

4.1 FPW and SPW

It is readily verified that in both FPW and SPW, losses happen with zero probability in equilibrium. Therefore, loss-averse equilibrium bidding coincides with risk-neutral equilibrium bidding if $\theta = 1$ and with risk-averse equilibrium bidding if $\theta < 1$.

Proposition 5. *Suppose only the winner's identity is revealed to the outside observer. Then, (i) equilibrium bidding in FPSB is not affected by loss aversion, (ii) equilibrium bidding in SPSB is* *not affected by loss aversion, and (iii) in equilibrium, expected revenue is the same in FPSB and SPSB if $\theta = 1$ and FPSB yields higher expected revenue than SPSB if $\theta < 1$.*

4.2 FPWP and SPWP

As previously in the case when only the winner identity is revealed, FPWP is not affected by loss aversion because losses happen with zero probability in the risk-neutral case. Now, consider SPWP. A bidder with a type v, pretending to be a type w, faces the following problem:

$$\max_{w} U(v,w) = \int_0^w (v - A(x) + \gamma V(x))^\theta \, dF^{(1)}(x) + \beta H(w)\big(\gamma L(w)\big)^\theta + \beta \int_w^{\bar{v}} \big(\gamma L(x)\big)^\theta \, dK(x)$$

The equilibrium follows from FOC:

$$\left.\frac{\partial U(v,w)}{\partial w}\right|_{w=v} = f^{(1)}(v)(v - A(v) + \gamma V(v))^\theta + h(v)\big(\gamma L(v)\big)^\theta + \beta\theta H(v)\big(\gamma L(v)\big)^{\theta-1}\gamma L'(v) - \beta(\gamma L(v))^\theta k(v) = 0.$$

For the parameters used in BGOT, it follows that

$$\left(\frac{5v}{4} + \frac{1}{4} - A(v)\right)^\theta = \beta\left(\frac{3}{8}\right)^\theta v^{\theta-1}(v(1+\theta) - \theta).$$

Note that the left side of the previous equality is negative for all $v < \frac{\theta}{1+\theta}$. It follows that

$$A(v) = \tfrac{5v}{4} + \tfrac{1}{4} - \beta^{1/\theta}\tfrac{3}{8} v^{\frac{\theta-1}{\theta}} (v(1+\theta) - \theta)^{1/\theta} \quad (3)$$

Let A_∞ the bidding strategy for SPWP in the case of infinitely loss-averse bidders, i.e., $A_\infty(v) = \lim_{\theta \to +\infty} A(v)$ for all $v \in [0,1]$. Equation (3) implies, for $v < 1$,

$$A_\infty(v) = \frac{7v}{8} + \frac{1}{4} < \frac{v}{2} + \frac{5}{8} = B(v).$$

It follows that loss aversion leads to lower bids than risk neutrality.

Proposition 6. *Assume* $\gamma = 1/2$, $n = 3$, $F = U[0,1]$. *If the winner identity and her payment are revealed to the outside observer, loss-averse bidders bid less aggressively than risk-neutral bidders in SPSB.*

Loss aversion captures the bidders' distaste to bid substantially more than their value as the equilibrium prediction dictates for risk-neutral bidders. These findings are consistent with the data in BGOT, where only 49.6% of the bidders bid above value and the majority of these do not bid as aggressively as predicted in the risk-neutral equilibrium. Proposition 6 implies that equilibrium revenue for loss-averse preferences is lower than in the risk-neutral case. The intuition is similar than the one for risk-averse preferences.

4.3 Revenue comparison

In this section, we compare expected equilibrium revenue assuming loss aversion bidders. Let $R^T_{\theta,\beta}$ denote the revenue for case T, i.e., FPW, FPWP, SPW, or SPWP, where bidders' degree of loss aversion is characterized by $\theta > 0$ and $\beta \geq 1$.

Compared to the risk-neutral case, loss aversion affects equilibrium bidding strategies negatively in SPSB, and therefore can lead to a lower revenue in the later. As a consequence, SPWP with sufficiently loss-averse bidders yields lower revenue than SPW and FPWP with risk-neutral bidders:

$$R^{SPWP}_{\infty,\beta>1} = \frac{11}{16} = 0.6875 < R^{SPW}_{1,1} = R^{SPW}_{\infty,\beta>1} \approx 0.72 < R^{FPWP}_{1,1} = \frac{3}{4}$$

and

$$R^{SPWP}_{\infty,\beta>1} = \frac{11}{16} < R^{SPW}_{\theta>0,\beta>1} < R^{SPW}_{1,1} \approx 0.72$$

for all (θ, β) such that $\beta^{1/\theta} < \frac{629}{280}$, where $R^{FPWP}_{1,1}$ and $R^{SPW}_{1,1}$ denote the expected revenue in the risk-averse equilibrium of FPWP and SPW. We conclude that for sufficiently loss-averse bidders, the revenue ranking of SPWP and SPW swaps compared to the risk-neutral case, qualitatively consistent with BGOT's data (see Table 1).

Proposition 7. *Assume* $\gamma = 1/2$, $n = 3$, $F = U[0,1]$. *For sufficiently loss-averse bidders, the revenue ranking of SPWP and SPW swaps compared to the risk-averse case, and are lower than FPWP.*

All in all, next to risk aversion, loss aversion is another potential candidate to explain several anomalous experimental results in BGOT.

5. Conclusion

This paper has offered the equilibrium results for risk-averse and loss-averse bidders with signalling concerns, i.e., bidders who care about how they are perceived by an outside observer. Our results are twofold. First, unlike in the usual auction settings, when the winner's identity and her payment are revealed, risk aversion and loss aversion yield less aggressive bidding behaviour in SPSB than in the risk-neutral case. Second, risk aversion and loss aversion are qualitatively consistent with several anomalies in Bos *et al.*'s (2021) experiment. Bos *et al.* (2021) find the ranking FPWP > FPW > SPW > SPWP while the ranking under risk-neutral bidders is SPWP > FPWP > FPW = SPW. In this paper, we have shown that both risk aversion and loss aversion may explain the ranking reversal observed in the experiment.

Our study may inspire new research analysing the effect of risk aversion and loss aversion in other settings than the ones studied in this paper. New research could examine auction formats other than the FPSB and SPSB, like the all-pay auction or the English auction, as well as other information disclosure policies than revealing who wins the auction and how much the winner pays, including revealing the winning bid (rather than winner's payment) or all bids. Such analysis would naturally fit the research agenda aimed at finding optimal auctions in settings where bidders have signalling concerns (Dworczak, 2020; Bos and Pollrich, 2022).

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