

Agreeing on efficient emissions reduction^{*}

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Abstract

We propose a simple mechanism providing incentives to reduce harmful emissions to their efficient level without infracting upon productive efficiency. The mechanism employs a contest creating incentives among participating nations to simultaneously exert efficient productive and efficient abatement efforts. Participation in the most stylised form of the scheme is voluntary and individually rational; all rules are mutually agreeable and are unanimously adopted if proposed. The scheme balances its budget and requires no principal. In a perhaps more realistic stochastic output version which could potentially inform policy decisions, we show that the transfers required by the efficient mechanism create a mutual insurance motive which may serve as effective rationale for the (gradual) formation of International Environmental Agreements. (JEL C7, D7, H4, Q5. Keywords: *Climate policy, Contests, Agreements.*)

1 Introduction

The disappointing series of failures to reach agreement among the 196 members of the United Nations Framework Convention on Climate Change in Copenhagen, Cancún, Durban, Doha, Warsaw,

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and Lima (2009–14) highlights the international impasse in preventing further global warming. Yet action seems to be called for: recent research reports shrinking ice mass balance from both Greenland and Antarctica with a projected sea-level rise of one to two meters by 2100.¹ Since an estimated 180 million people live currently in locations less than one meter above sea level, the impact of such a change on the world economy will be substantial.² This paper studies and answers two questions arising in this context: *i*) How can incentives be provided to reduce harmful emissions to their socially efficient levels while not infracting upon productive efficiency?³ *ii*) How can international agreement on the parameters of this or similar redistributive mechanisms be found?

In our simple model environment with additively separable cost of production and abatement, there are two ways to reduce emissions: by producing less or by abating more; there is no trade in inputs or outputs. The mechanism we propose plays on these two aspects in order to achieve efficiency in both: a stylised contest—based on a relative ranking of all nations’ abatement efforts—rewards the countries with the highest abatement efforts with some share of joint agreement output. In a nutshell, marginal production is ‘taxed’ to fund a prize pool, and marginal abatement increases the probability of winning a share of this pool. By designing the contest appropriately, both equilibrium incentives can be set efficiently at the margin. The precision with which the contest ranking is correct, i.e., the precision of mutual abatement monitoring, is one of the design parameters of our proposed mechanism.

In equilibrium, this efficiency inducing contest takes the form of a redistributive mechanism which returns some share of the collected tax pool in the form of prizes to the participants. As it turns out, for sufficiently volatile individual GDP, the variance of the redistributed income is lower than that of standalone individual income, even when the added randomness through the contest’s prize structure is taken into account. In this case, the redistributive contest mechanism can fulfill aspects of a mutual insurance agreement which can entice a country with a sufficiently strong dislike of income fluctuations to join an agreement on which, in the absence of this income smoothing argument, it would prefer to free ride.

¹ See, for instance, Dasgupta et al. (2009) or Allison et al. (2009). Mitrovica et al. (2009) predict less uniform sea level changes with a rise of up to 6.3 meters at some coastal sites in the northern hemisphere upon a total collapse of the West Antarctic Ice Sheet due to the loss of gravitational pull from this ice mass.

² The original estimate of 180 million is from Nicholls (1995); Hanson et al. (2011) estimate the economic effects of climate change on coastal cities and ports. A recent analysis of migration induced through climate change is, for instance, Kniveton et al. (2012).

³ Both productive and abatement efficiency are defined as the respective levels of efforts which maximise social welfare in the absence of information deficiencies or incentive aspects.

The main emissions type we have in mind for our model is greenhouse gases. These are widely seen as the main contributing factor to global warming. Emitted by one country as inherent part of the productive process, they are distributed around the globe regardless of where they were produced and, as such, present an externality. A reduction of emissions benefits all countries but the costs of such reductions are carried individually. This generates a classic free-rider problem in which each country would like the threat of global warming removed but none is ready to pay the cost.⁴ We think of the abatement efforts as the difference between ‘business as usual’ investments and green investments; the former are the investments that firms make without considering their environmental impact while green investments purposefully intend to reduce CO₂ emissions generated during production.

Nordhaus (2006), among others, advocates the implementation of market-based instruments and, more specifically, a world harmonised tax on each ton of CO₂ emissions, the revenue from which may be used at will by each national government. This proposal presents many advantages over the status quo; it can achieve efficient abatement at the world level, is simple to implement, and taxes are well-known instruments. However, imposing a new, harmonised tax (unilaterally) may be politically unattractive while the individualised winning incentives that our contest scheme presents make participation individually attractive. Moreover, for a tax to correct abatement incentives and simultaneously provide efficient production incentives, it needs to be complemented with a subsidy.⁵ This type of combined mechanism, however, will typically not balance its budget and will therefore give rise to further negotiations. In addition, as abatement efforts are difficult to measure in absolute terms, countries may be tempted to present productive investment as abatement efforts to get higher subsidies. In contrast, the informational requirements for a relative ranking of abatement efforts may be easier to satisfy than those for a cardinal scale. Moreover, a contest prize can be fixed independently of the competitors’ abatement efforts while fixed subsidies based on piece-rates depend on absolute levels. As our ranking is relative, it would not be affected by an overall abatement measurement inflation. Thus, a ranking allows to use indirect abatement measurements which could

⁴ One may advocate the view that some countries could climatically benefit from warming. Russian President Vladimir Putin, for instance, is reported to have said that climate change might be good for his country, as people would no longer need to buy fur coats (Reuters, 2-April-07). The impact on the world economy and consequences in terms of migration, however, make us pessimistic about the likelihood of emerging net beneficiaries.

⁵ The incentive scheme we propose is based on a verifiable, relative ranking of abatement efforts. We do not require the verifiability or cardinal measurement of these efforts. A subsidy, however, is likely to require both cardinal measurement and some verification mechanism.

be less easily manipulated than other, cardinal, measures entering tax calculations.

This paper makes two contributions. First, we show that a contest can implement efficiency in a specific environmental model along both productive and abatement effort dimensions. Second, we demonstrate that (gradual) agreement formation is possible in this model if nations are sufficiently averse to income-variance. Both results are theoretical. In addition, we provide a benchmark result showing which model-resources are required to implement the first-best solution under the following objectives: absence of a principal, efficiency (i.e., no distortions of the welfare maximising allocation) in both effort dimensions, and budget-balancing. Even if the highly stylised mechanism we discuss may seem unrealistic and difficult to implement directly, we hope that our analysis can deliver new and significant insights on the available options, on the cost of abatement, and on the implementation of agreement mechanisms.

Related literature

Public goods & contests. The most directly related studies of public goods provision problems relating to contests that we are aware of are Morgan (2000) and Giebe & Schweinzer (2014).⁶ Morgan (2000) studies a lottery mechanism which uses proceeds obtained from ticket sales for the provision of a public good. Contrasting with our analysis, he is neither concerned with designing a mechanism to implement exact efficiency nor with balancing the mechanism's budget. Using a fixed precision contest, he obtains the result that contests are unable to implement exact efficiency. Giebe & Schweinzer (2014) explore the possibility of the efficient provision of a public good through non-distortionary taxation of a private good which is linked to a lottery. By fine tuning the tax with the lottery, they are able to get efficient consumption levels for both private and public goods. This is in a similar spirit as the present analysis, but their individual public good contribution is only a function of the private good consumption and not at the individual's discretion. Moreover, our specific environmental model needs to balance two dimensions of inefficiency: excessive production and insufficient reduction of emissions. This is impossible in their single-dimensional model where

⁶ The idea that in many circumstances efficient efforts can be induced by awarding a prize on the basis of a rank order among competitors' efforts is due to Lazear & Rosen (1981). This insight has found numerous applications and extensions, for instance in the work of Moldovanu & Sela (2001), or Siegel (2009). For a detailed survey of the contests literature see the comprehensive Konrad (2008). The technically closest contest paper in this literature is Gershkov et al. (2009) who analyse the efficient effort choice in team-partnership problems.

tailoring only private goods consumption can lead to efficiency.⁷

Agreements. Our team setup is motivated by the generally accepted necessity for international environmental agreements (IEA) to be self-enforcing: there is no supranational principal to enforce such arrangements between countries. The consensus in the literature seems to be that there is no consistent theoretical basis on which large voluntary agreements can be formed among independent states which do not have to revert to exogenous reward, punishment or exclusion strategies to avoid free-riding on emissions reduction.⁸ The main contributions have found that IEA are either unlikely to consist of many participants, or if they do, are similarly unlikely to produce substantial benefits.

The basis for the risk-sharing argument which is the main underlying motivation for our players to voluntarily join an agreement is Wilson (1968) who discusses the optimal behaviour of individuals making a common decision under uncertainty that results in a payoff to be shared. As in our model, the strictly risk-averse players face individual payoff uncertainty which they pool in a team.⁹

Recent contributions to the literature on IEA-membership dynamics include Breton et al. (2010) and Harstad (2015). Referring to climate change agreements, the latter shows how short-term agreements may have adverse effects on countries' investments in green technology. Indeed, as Buchholz & Konrad (1994) and Beccherle & Tirole (2011) point out, anticipating negotiations can decrease the level of R&D and green investments.¹⁰

Plan of paper. Following the model definition in section 2 we present our main efficiency results in section 3. The model is closed using a simple, simultaneous agreement game based on deterministic output. A much more general (and realistic) participation argument is developed in section 4 for stochastic output. Finally, we present a short, intuitive example which conveys much

⁷ In several interesting papers, Buchholz et al. (2011), Gerber & Wichardt (2009), Gersbach & Winkler (2012) and Gersbach & Winkler (2011) develop ingenious efficient public goods provision mechanisms. Although we share important ideas with these papers, neither models productive efforts and therefore cannot consider the issue of efficiently balancing polluting overproduction with abatement efforts.

⁸ See Barrett (1994), Barrett (2003), Aldy & Stavins (2007), and Guesnerie & Tulkens (2009) for the main results and further discussion.

⁹ This idea found multiple notable extensions and applications, for instance, in the work of Banks et al. (2001), or Demange (2008). Deaton (1991) models the optimal intertemporal consumption behaviour of consumers whose labour income is stochastic over time. He shows that individual savings can act like a buffer stock, protecting individual consumption against shocks. The same idea has been used to model consumption-savings choices and insurance motives across consumers' life-cycles. The same variance-compression idea is present in Green (1987) who discusses tax-financed unemployment insurance and Arnott & Stiglitz (1991) who model voluntary, non-market insurance between households under moral hazard.

¹⁰ Many environmental papers employ contests to model lobbying activities; see, for example, Hurley & Shogren (1997), Heayes (1997), or, more recently, Kotchen & Salant (2011) and the references therein. The literature on environmental contest modelling of abatement incentives is, nevertheless, small. The only paper that we are aware of Dijkstra (2007).

of the intuition of our results in section 5. Several model extensions, examples, an alternative family of success functions, and the details of proofs are relegated to appendices A–D which are available as “online supplementary material.”

2 The symmetric model

There is a set $\mathcal{N}$ of $n \geq 2$ risk-neutral players. These players are symmetric in the basic model.¹¹ Each player $i \in \mathcal{N}$ exerts efforts along two dimensions: productive effort $e_i \in [0, \infty)$ and abatement effort $f_i \in [0, \infty)$. The abatement efforts need not, in principle, be verifiable.¹² We denote the full effort vectors by $\mathbf{e} = e_1, \dots, e_n$ and $\mathbf{f} = f_1, \dots, f_n$, respectively. The effort costs $c_e(e_i)$ and $c_f(f_i)$ are assumed to be strictly convex and zero for zero effort. Productive efforts generate strictly concave individual gains of $y(e_i)$ and cause strictly convex global emissions of $m(\max\{0, \sum_h e_h - \sum_h f_h\})$ —only depending on the difference between global productive and abatement efforts—of which player i suffers a known share s_i .¹³ Emissions are seen as an externality, a by-product (or factor) of production.¹⁴ We normalise $\sum_h s_h = 1$ which introduces a public bad team problem and summarise a player’s utility in the absence of any incentive mechanism as

$$y(e_i) - s_i m \left(\sum_{j \in \mathcal{N}} (e_j - f_j) \right) - c_e(e_i) - c_f(f_i). \quad (1)$$

Into this problem we introduce an incentive system based on a ranking of individual abatement efforts and award the top-ranked players prizes. The total prize pool P , from which these prizes are taken is defined as the sum of fraction $(1 - \alpha)$ of each participant’s individual income or output $y(e_i)$, i.e., $P = \sum_i (1 - \alpha)y(e_i)$. Thus, the incentive mechanism redistributes income and its budget balances by definition. The incentive mechanism awards $\beta^1 P$ to the winner, $\beta^2 P$ to the player coming second, and so on, with $\sum_h \beta^h = 1$.

¹¹ Our main results apply to the symmetric setting. Subsection 3.3 generalises the model to the asymmetric case; its workings are illustrated in several examples in appendix D.1 and D.2.

¹² We view the ranking introduced below as generated by some automaton or monitoring device (see also footnote 15). While agreement on this machine and verifiability of its readings are indispensable, the underlying efforts themselves need neither be observable nor verifiable. If we were to add a zero-mean noise term to output (without changing anything else) productive efforts could not be deduced from output either.

¹³ Requiring non-negative differences in the damage function $m(\cdot)$ ensures that abatement efforts cannot substitute for productive efforts. Since this requirement is fulfilled for most of our analysis, we redefine $m = m(\sum_h e_h - \sum_h f_h)$ and only make the non-negative argument explicit when necessary. (The implied non-differentiabilities play no role in our model.)

¹⁴ We can think of $c_f(f_i)$ as the cost of moving from the status quo to the targeted level of greenhouse gas (ghg) emissions. Stern (2006) argues similarly that the cost of stabilising ghg emission will be at about 1% of GDP per year compared to ‘business as usual.’ abatement is seen as a cost to the productive process. In the non-separable case, productive abatement has some concave benefit $\tilde{y}(f)$ which we omit from our formal model.

We assume that some noisy (partial) ranking of the players' abatement efforts is observable and verifiable. We interpret this ranking technology as arising from the agreement's monitoring of mutual abatement efforts. It is part of the mechanism the players need to agree on and gives player i 's probability $p_i^h(\mathbf{f})$ of being awarded prize h as a function of the imperfectly monitored abatement efforts of all participants. We write $p_i(\mathbf{f}) = (p_i^1(\mathbf{f}), \dots, p_i^n(\mathbf{f}))$ and assume that $p_i^1(\mathbf{f})$ is strictly increasing in f_i , strictly decreasing in all other arguments, equal to $1/n$ for identical arguments, twice continuously differentiable, and zero for $f_i = 0$ with at least one $f_{j \neq i} > 0$, $j \in \mathcal{N}$.¹⁵ Ties are broken randomly. While we consider this monitoring technology as endogenous to a realistic agreement formation process, we assume that this technology is already determined when the actual environmental contest game is played. Hence, we treat the contest's ranking precision $p(\mathbf{f})$ as exogenous to all matters dealing with the characterisation and existence of equilibria.

We use the above introduced interpretation of $p_i^h(\mathbf{f})$ as a probability of winning the full prize because this is the standard reading used in contest models. This is entirely equivalent, however, to the interpretation of $p_i^h(\mathbf{f})$ as the 'cardinal' *share* of the tax pool P that is allocated to player i in dependence of all players abatement efforts. Since this interpretation does not require the transfer of exorbitantly large payments to a 'winning' player, it might well be the more practically fruitful way of interpreting our model. Formally, as pointed out above, the two interpretations are equivalent.

Given a ranking $p(\mathbf{f}) = (p_1(\mathbf{f}), \dots, p_n(\mathbf{f}))$, a (subgame perfect) equilibrium in this contest game consists of two elements: an identity independent pair (α, β) specifying the tax and tournament prizes and a pair of efforts (e, f) determining output and winning probabilities. Since we are implementing efficiency we are looking for a symmetric equilibrium in pure strategies.¹⁶

¹⁵ Since this contest success function is general, the abatement efforts determining the contest outcome can be easily normalised with respect to, for instance, the individual (perceived) emission consumption share s_i . As usual, this ranking technology can be interpreted as monitoring technology, i.e., the slope of the function can be determined, e.g., by the frequency of inspections or the design of surveillance equipment. From a design point of view, the underlying assumption is that higher monitoring precision comes at a higher cost; infinite precision is not attainable. The inclusion of some monitoring cost ω financed out of the prize pool which is then split βP , $1 - \beta - \omega$, ω) is straightforward and does not qualitatively change any of our results.

¹⁶ The efficient allocation is symmetric because of the assumed concavity of production and cost convexity. Especially in the more complicated model variants discussed in the extensions, there may well be other (mixed) equilibria, perhaps of an asymmetric nature, which we disregard for the present analysis. The reason is that they can never implement the efficient allocation which is unique under our assumptions.

2.1 Timing and participation

This subsection defines a symmetric, simultaneous agreement formation game in order to derive the efficient mechanism in a simple setting. Note that a more general participation game supporting gradual agreement formation is defined in section 4 on the basis of a mutual insurance idea resulting from stochastic output.

Since the players' expected payoffs are symmetric in the basic model, we can think of a simple proposal game in which the identity independent design parameters $\langle \alpha, \beta; p(\mathbf{f}) \rangle$ are proposed by one randomly chosen player and the game is played if and only if all others simultaneously agree to the proposed parameters. The equilibrium concept used in such a game is subgame-perfect equilibrium which our solution satisfies. In order to discourage strategic disagreement, our design is slightly more involved: we propose a two-stage mechanism at the first stage of which an arbitrary player (called player 1) is randomly chosen to propose the two balanced budget contracts $C = \langle \alpha, \beta; p(\mathbf{f}) \rangle$ and $C' = \langle \alpha', \beta'; p(\mathbf{f})' \rangle$. The first contract C is invoked if all players agree to participate in the agreement. It implements efficient efforts in subgame perfect equilibrium. The second contract C' is invoked by the agreeing players if at least one player fails to participate and implements inefficient efforts which successfully deter non-agreement.¹⁷

More precisely, at the first stage of the game, if all players accept C , then the contest specified by C is set up, players commit their output shares $(1 - \alpha)y(e_i)$ and the game proceeds to the next stage. If at least one player rejects C , the agreeing players form a residual agreement, implement C' and again proceed to the second stage. If fewer than two players agree to setting up the mechanism (C, C') , the game ends and each player obtains their individual utility without agreement. At the second stage, conditional on the formation of an agreement, players choose their efforts simultaneously to maximise own expected utilities. The noisy ranking of abatement efforts specifies a winner, second, etc, final output realises, and the prize pool is redistributed to the winner, second, etc, according to the contract specified by C , or C' , respectively.

Non-participation in the agreement can be discouraged by either the simple simultaneous agreement game described above, or the threat of agreement members to implement C' . It is easy to see that such a sufficiently strong punishment contract C' always exists: setting $C' = \langle \alpha' = 1, \cdot; \cdot \rangle$

¹⁷ Formally, this second problem is equivalent to allowing a signatory to exit the agreement (i.e., renege on his commitments) after the agreement is formed. As pointed out by Chander & Tulkens (2009), this contract will typically not be renegotiation proof and commitment to C' is crucial. We discuss alternative enforcement measures in appendix A.1 and a general agreement model based on stochastic output in section 4.

replicates the pre-agreement scenario in which all players are worse off than with an agreement.¹⁸ This extreme form of punishment, however, will typically not be necessary. As illustrated in subsection 3.2 and example D.3 in the appendix, a second-best contract C' will generally be able to implement higher levels of abatement than those materialising absent an IEA.¹⁹

2.2 First-best benchmark

Much of the economics behind our results can be understood from the simple symmetric two players case on which the main body of the paper rests. An intuitive two player example can be found in section 5; it conveys most of the intuition while involving only minimal technicalities. For this two-player setup, we label players as i, j with $i = 1, 2$ and $j = 3 - i$. We define the efficient levels of both productive and abatement efforts (e^*, f^*) as those maximising social welfare absent of incentive aspects²⁰

$$\begin{aligned} \max_{(e, f)} u(e, f) &= 2y(e) - m(2e - 2f) - 2c_e(e) - 2c_f(f) \\ \frac{\partial u}{\partial e}, \frac{\partial u}{\partial f} &\Leftrightarrow \begin{cases} y'(e^*) = m'(2e^* - 2f^*) + c'_e(e^*), \\ m'(2e^* - 2f^*) = c'_f(f^*). \end{cases} \end{aligned} \quad (2)$$

In the absence of an incentive scheme, a player $i = 1, 2$ individually maximises

$$\begin{aligned} \max_{(e_i, f_i)} u_i(e_i, f_i) &= y(e_i) - s_i m(e_i + e_j - f_i - f_j) - c_e(e_i) - c_f(f_i) \\ \frac{\partial u}{\partial e_i}, \frac{\partial u}{\partial f_i} &\Leftrightarrow \begin{cases} y'(e) = s_i m'(2e - 2f) + c'_e(e), \\ s_i m'(2e - 2f) = c'_f(f). \end{cases} \end{aligned} \quad (3)$$

in which s_i is player i 's local share of global emissions. We write $e = e_i = e_j$, $f = f_i = f_j$ after maximisation. Since we normalise $s_i + s_j = 1$, the individual first-order conditions in (3) cannot both equal those in (2). In order to overcome this inefficiency in *both* dimensions, we introduce an endogenised rank-order emissions reduction reward scheme, i.e., a contest. We ask each participating nation to commit to contributing a share $(1 - \alpha)$ of their individual output $y(e_i)$ to the mechanism

¹⁸ For a detailed study of how punishments can be used to force agreement see Chander & Tulkens (1995).

¹⁹ Designing C' just sufficiently bad to serve as a deterrent resembles the idea of γ -core stability in Chander (2007). An alternative way of deterring this kind of free-riding on the agreement is to grant most favoured 'green' trading terms only to participating nations. This idea is further explored in appendix A.1. For a simulation of agreement stability using plausible data based on an integrated assessment model see Bosetti et al. (2013).

²⁰ The expressions following the curly bracket are the necessary first-order conditions for optimality. In the social planner's problem, these are also sufficient resulting from the concavity/convexity assumptions we make. The existence of equilibrium in our proposed mechanism will be discussed in proposition 2.

and therefore form a pool of prize money of size $P = (1 - \alpha)(y(e_i) + y(e_j))$. In a contest specifying player i 's winning probability as $p_i(\mathbf{f})$ based on both players' abatement efforts, we want to assign βP to the winner and $(1 - \beta)P$ to the player coming second. Notice that such a mechanism redistributes income. The individual problem under our incentive mechanism is therefore²¹

$$\max_{(e_i, f_i)} \underbrace{\alpha y(e_i)}_{\text{retained output}} + \underbrace{p_i(\mathbf{f})\beta P}_{\text{first prize}} + \underbrace{(1 - p_i(\mathbf{f}))(1 - \beta)P}_{\text{second prize}} - \underbrace{s_i m(e_i + e_j - f_i - f_j)}_{\text{damage from emissions}} - \underbrace{(c_e(e_i) + c_f(f_i))}_{\text{effort costs}}.$$

We define individual rationality as the requirement that the utility from participating in this mechanism for appropriately chosen $\langle \alpha, \beta; p(\mathbf{f}) \rangle$ exceeds *i*) the utility from non-formation of the agreement (3), *ii*) of free-riding on the others' abatement efforts *within* the agreement and *iii*) on free-riding on the others' abatement efforts *outside* the agreement. In the first case, no agreement exists at all while in the third case, an agreement outsider benefits from the abatement efforts of the agreement members. The second case concerns an agreement member's inefficient effort provision with committed output share.

3 The deterministic output model

3.1 Equilibrium characterisation and existence

We begin by characterising the design parameters which induce efficiency in our model. Recall that, under the contest scheme, an individual participant $i = 1, 2$ chooses a pair of efforts (e_i, f_i) to

$$\max_{(e_i, f_i)} \alpha y(e_i) + p(\mathbf{f})\beta P + (1 - p(\mathbf{f}))(1 - \beta)P - s_i m(e_i + e_j - f_i - f_j) - c_e(e_i) - c_f(f_i) \quad (4)$$

in which $p(\mathbf{f})$ is the probability of coming first in a ranking of abatement efforts f and the prize pool is $P = (1 - \alpha)(y(e_1) + y(e_2))$. We require that $y' > 0$, $y'' < 0$, $m' > 0$, $m'' > 0$, and $c'_{1,2} > 0$, $c''_{1,2} > 0$. We moreover assume that $m(\cdot)$ only depends on the difference of total productive minus abatement efforts. Taking derivatives with respect to both effort types, we obtain the simultaneous pair of first-order conditions defining individually optimal efforts (e_i, f_i) as

$$\begin{aligned} c'_e(e_i) + s_i m'(e_i + e_j - f_i - f_j) &= (1 - \alpha)(1 - \beta)(1 - p(f_i, f_j))y'(e_i) \\ &\quad + (1 - \alpha)\beta p(f_i, f_j)y'(e_i), \end{aligned} \quad (5)$$

$$c'_f(f_i) + p'(f_i, f_j)(1 - \beta)P = s_i m'(e_i + e_j - f_i - f_j) + p'(f_i, f_j)\beta P.$$

²¹ In the two-players setting, note that $p_i(\mathbf{f}) = 1 - p_j(\mathbf{f})$, $\beta^1 = \beta$ and $\beta^2 = 1 - \beta$.

Assuming tentatively that a symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$, $s_i = 1/2$ exists (until existence is demonstrated in proposition 2) this simplifies to

$$\begin{aligned} 2c'_e(e) + m'(2e - 2f) &= (\alpha + 1)y'(e), \\ 2c'_f(f) - m'(2e - 2f) &= 4(1 - \alpha)(2\beta - 1)p'(f, f)y(e). \end{aligned} \quad (6)$$

Equating these efforts to the efficient efforts e^* , f^* resulting from the solution to the social planner's problem in (2), we obtain

$$4p'(\mathbf{f}^*)(2\beta - 1) = \frac{y'(e^*)}{y(e^*)} \Leftrightarrow \begin{cases} c'_e(e^*) = \alpha y'(e^*), \\ c'_f(f^*) = 4(1 - \alpha)(2\beta - 1)p'(\mathbf{f}^*)y(e^*) \end{cases} \quad (7)$$

in which $\mathbf{f}^* = (f^*, f^*)$. We know from (2) that there exists an $\alpha \in [0, 1]$ to satisfy the first equation. Substituting this α into the second equation determines $\beta \in [1/2, 1]$ for a suitably chosen ranking $p(\cdot)$. Without further restrictions on the design parameters—and in particular the slope of the ranking technology $p(\cdot)$ in equilibrium—the set of necessary conditions in (7) can always be satisfied. Taking equilibrium existence as given (until we verify it in proposition 2), the following proposition establishes the precise criteria on the parameters for both productive and abatement efficiency to obtain simultaneously for any number of players $n \geq 2$. In all following results we employ the simple prize structure $\beta = \left(\beta^1, \frac{1-\beta^1}{n-1}, \dots, \frac{1-\beta^1}{n-1}\right)$ assigning a single winning prize and another prize to all losers. This is not necessary but simplifies the exposition considerably.

Proposition 1. *For appropriately chosen $\langle \alpha, \beta; p(\mathbf{f}) \rangle$ and $P = (1 - \alpha) \sum_j y(e_j)$, player $i \in \mathcal{N}$ chooses efficient productive as well as abatement efforts (e^*, f^*) in*

$$\max_{(e_i, f_i)} \alpha y(e_i) + \sum_h (\beta^h p_i^h(\mathbf{f}) P) - s_i m \left(\sum_h (e_h - f_h) \right) - c_e(e_i) - c_f(f_i). \quad (8)$$

The proof can be found in appendix B. As in (7), the main idea is to insert the efficient efforts into the first-order conditions of the incentive game and solve the resulting system for the mechanism's design parameters $\langle \alpha, \beta; p(\mathbf{f}) \rangle$. Proposition 3 below shows that a simple symmetric model in which every player decides simultaneously on whether to set up an agreement or not results in both unanimous agreement and full participation in symmetric, subgame perfect equilibrium.

A consequence of this first result is that full efficiency in the symmetric n -player model can be obtained with just two different prizes: one for the winner and another for everyone else. As one only needs to check for a winning 'abatement-champion,' such a scheme is easy to monitor. Since

the general objective (8) is not necessarily well-behaved without further assumptions on $p(\cdot)$, we proceed to show that equilibria exist for the subclass of problems governed by the Tullock success function $p_i(f) = f_i^r / (f_1^r + \dots + f_n^r)$. Hence, for the following proposition we concern ourselves with mechanisms of the form $\langle \alpha, \beta; r \rangle$, in which the designer may choose the parameter r (interpreted as monitoring intensity) as seen fit but subject to equilibrium existence. Depending on this exponent r , the Tullock function may be first convex and then concave, resulting in the underlying optimisation problem to be non-concave.

Proposition 2. *Consider a mechanism $\langle \alpha, \beta; r \rangle$. Under the Tullock success function, if c_f is sufficiently convex, a symmetric pure-strategy equilibrium exists in which production and abatement are efficient.*

The proof of this proposition establishes a sufficient condition for quasi-concavity of the players' objective. The main idea is to 'iron out' the non-concavity of the objective introduced through the success function by choosing a sufficiently convex cost function. The sufficient threshold OA:(15),²² derived in appendix B, ensures the existence of a symmetric pure-strategy equilibrium for contests $\langle \alpha, \beta; r \rangle$ governed by the Tullock success function by specifying an upper bound on admissible r . If this condition is respected, an equilibrium implementing the efficient efforts characterised in proposition 1 is certain to exist. Since r can be chosen by the agreement, this condition can in principle always be satisfied. If, however, for an exogenously given environment, the effort cost of abatement are insufficiently convex or, equivalently, the chosen monitoring precision r (or the equivalent slope of $p(f)$) is too high, then pure-strategy equilibria fail to exist. In that case, giving up the simple 'flat-loser' prize structure $\beta = \left(\beta^1, \frac{1-\beta^1}{n-1}, \dots, \frac{1-\beta^1}{n-1} \right)$ in favour of a structure which awards multiple first prizes $\beta' = (\beta^1 = \beta^2 \geq \dots \geq \beta^n)$ eases the existence problem at the expense of the implemented abatement efforts.²³

3.2 Agreement formation

In the symmetric and simultaneous case, the argument for full participation is almost trivial since every player's utility is identical and there is no improvement for any player if the agreement is

²² We identify equation (n) in the online appendix using the reference OA:(n).

²³ It is easy to see why this is the case: an equilibrium in which every player gets the same prize must exist (with zero abatement efforts). By continuity, equilibria (with small abatement efforts) will exist under a prize structure which gives the same prize to every player except the one coming last. For details see Schweinzer & Segev (2012).

not unanimously formed. Equilibrium existence then implies that free-riding on the abatement effort is not attractive once a nation is committed to the agreement.²⁴ However, as the number of participants in the mechanism goes up, the utility from free-riding on an existing agreement increases as the disutility from pollution $m(\sum_h(e_h - f_h))$ approaches the efficient level. Hence, if gradual agreement formation is allowed, then the only leverage left in the efficient contract C is the contest on the pre-committed output share $(1 - \alpha)$ —which is generally not sufficient to deter free-riding on an existing agreement. The contract C' is, however, capable of eradicating all gains from free-riding on the agreement by—in its most extreme form—replicating non-agreement pollution levels.

Proposition 3. *Participation in the symmetric mechanism specifying the pair of contracts $C = \langle \alpha^*, \beta^*; p^*(\mathbf{f}) \rangle$ determined through (8) and $C' = \langle \alpha' = 1, \beta' = 1/2; \cdot \rangle$ is individually rational in the sense that the utility from free-riding efforts e^s, f^s on C' cannot exceed the utility obtained when agreeing to C*

$$y(e_i^s) - s_i m(e_i^s + (n-1)e'(\alpha', \beta', \cdot) - f_i^s - (n-1)f'(\alpha', \beta', \cdot)) - c_e(e_i^s) - c_f(f_i^s) \leq u_i(e^*, f^*). \quad (9)$$

This result is intuitive as the agreement parameters are identity independent and the efficient allocation maximises welfare. The symmetric agreement will therefore always be formed. As participation in the agreement is individually rational, free-riding is fully deterred. Off the equilibrium path, the second-best contract C' —which is implemented if at least one player fails to participate—may still allow substantial emissions reductions. Our example D.3 shows that the agreement's *raison d'être* needs not necessarily be surrendered to holdup attempts. Further (ad-hoc) ways in which agreement participation can be ensured in the deterministic output version of our model, i.e., granting most favoured 'green' trading terms only to participating nations and environmental certification, are illustrated in appendix A.1.

3.3 The asymmetric model

In order to show that our efficiency result is not an artifact of our symmetry assumptions, this subsection explores cost asymmetries among players. As in general asymmetric contracting problems, the efficient asymmetric mechanism needs to be identity dependent, i.e., the simple symmetric

²⁴ Once a nation has committed her share of output $(1 - \alpha)y(e_i)$ to the agreement, the only possibility for free-riding is on her abatement efforts—which we show in proposition 1 to be suboptimal. A deposit-based mechanism à la Gerber & Wichardt (2009) would rectify any (in our paper unmodeled) 'paying-up' problems among committed agreement members.

mechanism components (α, β) now need to be designed for each individual player in the form (α_i, β_i) , $i \in \mathcal{N}$. This is interpreted as the requirement for each player to pay an individualised tax rate α_i and obtain a share $\beta_i P$ in case of winning (resulting in losers' shares which depend on the identity of the winning player).

Let $i \in \mathcal{N}$ and $n \geq 2$. The following result shows that, for appropriately chosen $\langle \alpha_i, \beta_i; p(\mathbf{f}) \rangle_i^n$, prize pool $P = \sum_{j=1}^n (1 - \alpha_j) y_j(e_j)$, and prize structure $(\beta_i, \frac{1-\beta_i}{n-1}, \dots, \frac{1-\beta_i}{n-1})$, efficient solutions exist to player i 's asymmetric problem

$$\max_{(e_i, f_i)} \alpha_i y_i(e_i) + p_i^1(\mathbf{f}) \beta_i P + \sum_{j \neq i} p_j^1(\mathbf{f}) \left(\frac{1 - \beta_j}{n - 1} \right) P - s_i m \left(\sum_{i=1}^n e_i - f_i \right) - c_{i,e}(e_i) - c_{i,f}(f_i). \quad (10)$$

Analogous to (2), let player i 's asymmetric efficient efforts be given by

$$y'_i(e_i^*) = m'(G) + c'_{e_i}(e_i^*) \text{ and } m'(G) = c'_{f_i}(f_i^*) \quad (11)$$

in which $G = \sum_{j=1}^n (e_j - f_j)$.

Let the payment shares α_i and winning shares be identity-dependent, i.e., a winning player i gets share β_i and a winning j gets share β_j of the total prize pool $P = \sum_{j=1}^n (1 - \alpha_j) y_j(e_j)$. Thus, taking all e_j^*, f_j^* , $j \neq i$, as given by (11), player i maximises (10). Taking derivatives with respect to e_i, f_i and inserting (11), determines player i 's best response through

$$\alpha_i = \frac{y'_i(e_i)(1 - H_i) - (1 - s_i)m'(G)}{y'_i(e_i)(1 - H_i)}, \text{ where } H_i = \beta_i p_i^1(\mathbf{f}) + \frac{\sum_{j \neq i} (1 - \beta_j) p_j^1(\mathbf{f})}{(n - 1)}, \quad (12)$$

$$\beta_i = \frac{(n - 1)((1 - s_i)m'(G)) - \sum_{j \neq i} (1 - \beta_j) p_{j(f_i)}^1(\mathbf{f}) P}{p_{i(f_i)}^1(\mathbf{f})(n - 1) P}$$

and $p_{i(f_i)}^1(\mathbf{f})$ denotes $\frac{\partial}{\partial f_i} p_i^1(\mathbf{f})$. The mechanism (12) elicits asymmetric efficient efforts (e_i^*, f_i^*) . Without putting any restrictions on the numbers α_i and β_i , a (numerical) solution to (12) can always be found. Since the same is true for the best responses of player i 's opponents, we confirm that a solution to the complete system $(\alpha_1, \dots, \alpha_n; \beta_1, \dots, \beta_n)$ exists.

Proposition 4. *A solution to the asymmetric system (12) exists.*

The proof relies on a fixed point argument to demonstrate that a solution to the asymmetric problem exists. For the asymmetric environment, we can provide a complete, analytic characterisation of the asymmetric abatement contest only for specific functional forms. For an exhaustive illustration that and how this is possible, see section D.1 in the appendix.

3.4 Asymmetric players with relative per-unit-GDP abatement efforts

This section indicates how to extend the above asymmetric model to an asymmetric contest based on *relative* abatement efforts per-unit-GDP. This alternative to the standard model incorporates an equally ‘fair’ measure of abatement effort for all asymmetric contestants. This asymmetric contest takes the form

$$u_i(\mathbf{e}, \mathbf{f}) = \underbrace{\alpha_i y(e_i)}_{\text{output}} + \underbrace{p_i^1(\mathbf{e}, \mathbf{f}) \beta_i P}_{\text{winning}} + \underbrace{\sum_{j \neq i \in \mathcal{N}} p_j^1(\mathbf{e}, \mathbf{f}) \frac{1 - \beta_j}{n - 1} P}_{\text{losing}} - \underbrace{s_i m \left(\sum_h (e_h - f_h) \right)}_{\text{damage}} - \underbrace{c_i(e_i, f_i)}_{\text{cost}}, \quad \text{for the redistribution pool } P = \sum_{i=1}^n (1 - \alpha_i) y(e_i), \quad (13)$$

in which the winning probability $p_i^1(\mathbf{e}, \mathbf{f})$ is a two-dimensional version of the generalised Tullock success function. In particular, the winning probability $p_i^1(\mathbf{e}, \mathbf{f})$ is now based on the ratio x of the two strategic variables: abatement efforts over a function of productive efforts²⁵

$$p_i^1(\mathbf{e}, \mathbf{f}) = \frac{x_i^r}{x_1^r + \dots + x_n^r}, \quad x_i = \frac{f_i}{y(e_i)}. \quad (14)$$

The probabilities $p_{j \neq i}^1(\mathbf{e}, \mathbf{f})$ are defined in the same way as player j ’s probability of winning in a contest not involving player i . We again use identity dependent tax rates α_i and winning shares β_i . For simplicity, we again only discriminate between winners and losers, i.e., if player i wins, then we award $\beta_i P$ to the winner and $\frac{1 - \beta_i}{n - 1} P$ to each of the losing players.²⁶

The contest success function employed in this subsection is therefore the following multi-dimensional version of the generalised Tullock success function²⁷

$$p_i^1(\mathbf{e}, \mathbf{f}) = \frac{(f_i / y(e_i))^r}{(f_1 / y(e_1))^r + \dots + (f_n / y(e_n))^r} = \frac{x_i^r}{x_1^r + \dots + x_n^r}, \quad r > 0. \quad (16)$$

Example D.2 in the appendix illustrates that efficiency can again be obtained in this mechanism using a numerical example.

²⁵ For completeness, we define that $p_i(\mathbf{e}, \mathbf{f}) = 1$ if $y(e_i) = 0$, $f_i > 0$ and all $y(e_{-i}) > 0$. Similarly, we let $p_i(\mathbf{e}, \mathbf{f}) = 1/m$ if $m = |y(e_j) = 0|_{j \in \mathcal{N}}$.

²⁶ This particular prize structure is not chosen for its normative appeal but for analytical convenience. Any other monotonic prize structure would be equally acceptable without qualitatively changing our results.

²⁷ To exclude unbounded ratios x_i , we assume that $p_i^1(\mathbf{e}, \mathbf{f}) = 0$ if $e_i = 0$. This discontinuity at zero plays no role in the examples discussed in this setup. The idea can be easily generalised to more than two dimensions. A simple way of achieving this is to use

$$\tilde{x}_i = \frac{f_i}{y_i^1(e_i^1) + \dots + y_i^m(e_i^m)} \quad (15)$$

in which $e_i^1, \dots, e_i^m$ is player i ’s m -dimensional ‘normalisation’ effort transformed, if necessary, by the functions $y_i^h(\cdot)$, $h = 1, \dots, m$. To the best of our knowledge, this ‘normalised,’ relative efforts ‘per-unit-GDP’ formulation of the Tullock success function is original to this paper. The general characterisation of this contest is irrelevant for the purposes of illustrating our argument and can be obtained from the authors upon request.

Although we demonstrate most of our results in a simplified symmetric setup, our asymmetric discussion above together with the examples in the appendix suggest that at least some of the attractive properties of the mechanism we introduce carry over to the asymmetric case. Finally, the question whether and in how far the efficient asymmetric abatement levels can be used directly as inputs into the contest success function or not is, of course, politically charged. We therefore restrict ourselves to pointing out that any standard normalisation of input efforts is feasible. For instance, it is perfectly possible to normalise inputs such that each country which exerts its efficient abatement level has the same equilibrium chance of winning the first prize $1/n$. An interesting implementation of this normalisation idea is to assign a vector of individual weights $\psi = (\psi_1, \dots, \psi_n)$, with each $\psi_i > 0$, to the heterogeneous contestants' abatement efforts turning the basic success function into

$$p_i(f, \psi) = \frac{\psi_i f_i^r}{\sum_j \psi_j f_j^r}, \text{ in which } \frac{\psi_i}{\delta_i^r} = \frac{\psi_j}{\delta_j^r} \forall j \neq i \quad (17)$$

and thus 'levelling the playing field.'²⁸ Following the assignment of ψ , one can proceed with the analysis of section 3 without further change. Thus, a strength of a contest-based mechanism is its ability to adopt different success functions: our concept of abatement effort can incorporate fairness considerations by encompassing, e.g., population, geographic size, or GDP.

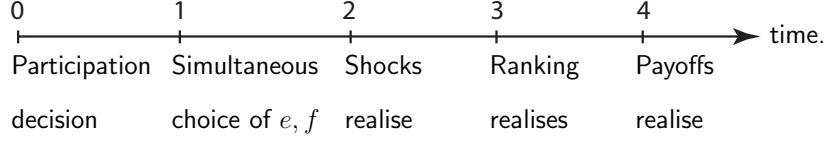
4 The stochastic output model

Since the section 3.2 suggests that we cannot generally secure participation of an outsider in an existing agreement without recurring to punishments or other renegotiation-prone (ad-hoc) measures, this section exploits the fact that the efficient mechanism introduced in the previous sections has redistributive properties. This allows us to derive a stronger, more methodical argument for agreement formation. In order to capture the mutual insurance idea of redistributive agreements, we make two key modifications to the deterministic framework of the model section 2: we allow for stochastic output y and consider risk-averse decision makers. In particular, we assume that a nation's output process is stochastic, i.e., given by $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, in which $\tilde{y}(e_i)$ is weakly concave and the shocks ε_i are distributed according to the law $\mathcal{L}(\mu = 0, \sigma^2)$ characterised by the mean μ and the finite variance σ^2 of some continuously differentiable distribution F with symmetric

²⁸ This idea of creating a more symmetric contest through the appropriate choice of ψ was studied recently, among others, by Franke et al. (2011), and Franke (2012).

probability density F' over (any subset) of $(-\infty, \infty)$.²⁹ As suggested by, e.g., Chamberlain (1983) or Owen & Rabinovitch (1983), any member of the class of elliptical distributions is eligible for this distribution F .³⁰

The full timing of the modified interaction is



4.1 Preferences and information

Expected utility functions are defined over a decision maker's uncertain wealth w . The idea of risk aversion is incorporated in the curvature of a Bernoulli-utility function v over certain payoffs. We assume that this v is concave and that all players' embodied attitude towards risk is identical. The only stochastic influence in our model comes from idiosyncratic shocks to output. We adopt the following formulation for player $i \in \mathcal{N}$

$$\begin{aligned} & \mathbb{E}[u_i(\mathbf{e}, \mathbf{f}; \varepsilon_i)] \\ &= \int_{-\infty}^{\infty} v(\underbrace{\alpha y(e_i, \varepsilon_i) + \sum_h (\beta^h p_i^h(\mathbf{f}) P) - s_i m \left(\sum_{i \in \mathcal{N}} (e_i - f_i) \right) - c_e(e_i) - c_f(f_i)}_{\text{risk-neutral wealth}}) dF(\varepsilon_i) \end{aligned} \quad (18)$$

with Bernoulli utility function $v(0) = 0$, $v' > 0$, and $v'' \leq 0$ and $P = (1 - \alpha) \sum_j y(e_j, \varepsilon_j)$. Since the player's choice of efforts are invariant under any increasing, concave transformation of v , we can split the optimisation stage—giving the choice of efforts—from the risk-based participation decision. Hence, methodologically, we start by analysing the above underbraced decision problem in isolation—this step is equivalent our risk-neutral analysis in section 3—and discuss the risk transformation separately in this section.

²⁹ The additive structure of the shock is crucial for the arguments we develop. We keep the stochastic process generating income uncertainty as general as possible in terms of distribution and variance while the mean is kept at zero for modelling convenience (otherwise the riskless model could not be used as a benchmark). For a combined flow & stock interpretation of income in our one-shot model this may not be unreasonable. A financial crisis, for instance, could be interpreted as a negative shock to this composite variable.

³⁰ Elliptical distributions are a generalisation of the normal family containing, among others, the Student-t, Logistic, Laplace and symmetric stable distributions. A detailed presentation of these distributions is available in Landsman & Valdez (2003) and Fang et al. (1987).

4.2 The efficient stochastic mechanism

Our results on the stochastic output mechanism show that, ceteris paribus, nations endowed with variance-averse preferences favour the redistributive contest over standing alone. The idea behind this variance compression in the agreement is easiest to see for independently distributed stochastic output of the form $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, i.i.d. $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$. We start with the general participation result for the contest game discussed in section 3.1 in this i.i.d. framework.

4.2.1 The contest mechanism

Proposition 5. *Consider individual output $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, $i \in \mathcal{N}$, for independently and identically distributed $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$. Then the equilibrium individual payoff variance of the symmetric, balanced budget contest mechanism $\langle \alpha^*, \beta^*; p(\mathbf{f}) \rangle$ characterised in proposition 1 is lower than the standalone payoff variance provided that standalone output variance σ^2 is sufficiently high.*

The threshold condition identified in the proof of the previous proposition is satisfied whenever there is a sufficiently high standalone income variance σ^2 which the redistribution mechanism can compress. If σ^2 exceeds this threshold, then the insurance arising from pooling individual output outweighs the payoff variance introduced by the contest. In our standard example for the two players case and for $r = 2$, squared output $\mathbb{E}[y(e, \varepsilon)^2] \simeq .28$ and our sufficient condition implies that the redistributed payoff variance is below individual output variance whenever $\sigma^2 \geq 0.0039$, i.e., for any reasonable variation. (This is demonstrated in example D.4 in the appendix.) As the threshold is increasing in the number of agreement participants n , however, the condition gets harder to satisfy for larger agreements. Hence, the theory predicts that multiple smaller agreements are easier to sustain than one large agreement.

We now extend the previous result to the case where the covariance between individual shocks is positive. For reasons of tractability we assume identical covariance between all pairs $\text{Cov}(\varepsilon_i, \varepsilon_j) = \bar{\sigma}^2$, $i \neq j \in \mathcal{N}$. Then the following result provides a sufficient condition for the possibility of variance compression.

Proposition 6. *Consider individual output $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, $i \neq j \in \mathcal{N}$, for identically distributed $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$ with $\text{Cov}(\varepsilon_i, \varepsilon_j) = \bar{\sigma}^2$. Then the equilibrium individual payoff variance of the symmetric, balanced budget contest mechanism $\langle \alpha^*, \beta^*; p(\mathbf{f}) \rangle$ characterised in proposition 1 is*

lower than the standalone payoff variance provided that standalone output variance σ^2 is sufficiently higher than the uniform covariance $\bar{\sigma}^2$.

The idea behind the proof is that whenever individual outputs are not perfectly aligned, a redistributive agreement can compress the variance of individual payoff. The resulting threshold condition is, however, more demanding than the corresponding condition in the i.i.d. environment.

4.2.2 The sharing mechanism

This subsection shifts attention from the contest interpretation of our mechanism to the sharing interpretation in which $p_i(\mathbf{f})$ is seen as a deterministic, endogenous share of the tax pool that is allocated to player i in dependence of all players abatement efforts. This interpretation should be applicable to a much wider class of existing agreements than the precise contest mechanism analysed in the previous subsection. Under this interpretation, there are no ‘winning’ or ‘losing’ players and each player gets the share specified by $p_i(\mathbf{f})$ bounded from above by βP and from below by $(1 - \beta)P$. For this purpose, in contrast to the contest interpretation, abatement efforts $\mathbf{f}$ need to be contractible to some extent. Then, while the incentives incorporated in the mechanism ensure efficient efforts along both dimensions, all symmetric equilibrium transfers cancel out and there is no contest variance that adds to the participation problem. In this simpler setting, all we require for the general participation result is that not all ε_i , $i \in \mathcal{N}$, exhibit perfect positive correlation.

Proposition 7. *Consider individual output $y(e_i, \varepsilon_i) = \tilde{y}(e_i) + \varepsilon_i$, $i \in \mathcal{N}$, for identically distributed $\varepsilon_i \sim \mathcal{L}(\mu = 0, \sigma^2)$ with covariance σ_{ij} for $(\varepsilon_i, \varepsilon_j)$. Then any balanced budget mechanism which is redistributive, i.e., $\alpha^* < 1$ and symmetric, i.e., assigns equal winning probabilities in symmetric pure strategy equilibrium, has a lower variance than individual output.*

The main idea of our participation argument is that this reduction in income risk can be used to motivate the formation of an agreement. Although we study exclusively incentives for joining an international environmental agreement in this paper, the same basic variance compression argument should be applicable to many other international bodies. In order to show the versatility of the idea, we now turn to the analysis of the asymmetric redistributive mechanism.

The next proposition verifies the variance-compression intuition developed for the symmetric

redistributive pool for the asymmetric case under both independent and non-independent shocks. Again, we assume that shocks ε_1 and ε_2 are not perfectly positively correlated.

Proposition 8. *Consider a two-player, balanced budget sharing mechanism which is redistributive, i.e., $\alpha_1 < 1$, $\alpha_2 < 1$, and asymmetric, i.e., assigning not necessarily equal winning probabilities $p_1(e, f), p_2(e, f)$ in asymmetric pure strategy equilibrium. The equilibrium payoff from this class of mechanisms has a lower variance than individual output if and only if the following conditions are fulfilled for all $(i, j) \in \{1, 2\}$*

$$\begin{aligned} 1 &> \beta_i^2(1 - \alpha_j)^2 + (\beta_i - \alpha_i\beta_i - \alpha_i)^2 + 4\alpha_i\beta_i(1 - \alpha_i) \text{ for i.i.d. shocks,} \\ 1 &> \beta_i(3 - 2\alpha_i - \alpha_j)(2\alpha_i + \beta_i - \beta_i\alpha_j) + (\beta_i - \alpha_i\beta_i - \alpha_i)^2 \text{ otherwise.} \end{aligned} \tag{19}$$

The condition for variance compression depends on there being some redistribution in the first place: $\alpha_1^* + \alpha_2^* < 2$. Moreover, since we are only looking at two players, each $\alpha_i^* < 1$ in order to allow for risk pooling. From the point of view of player $i = 1, 2$, her winner's share $\beta_i^* > 1/2$ determines the income transfer in case of winning and $1 - \beta_j^*$, $j = 3 - i$, determines how much income is redistributed in case she loses. For asymmetric winning probabilities $p_i(e, f)$, the interplay of these variables in (19) determines when risk-pooling is possible.

Equations (19) look more complicated than they are. The reason is that we want to specify parameters (α, β) in sufficient generality to be applicable for any redistribution problem. Indeed, the conditions mean that our efficient mechanism leads to a lower variance than individual output if and only if the pair of parameters (α_i, β_i) for $i = \{1, 2\}$ are small enough. To provide an intuition, remark that if $\alpha_i \rightarrow 0$ and $\alpha_j \rightarrow 0$, our mechanism is better in terms of variance reduction if and only if $\beta_i, \beta_j \in [0, \sqrt{6}/2]$. Moreover our mechanism is still better if $\beta_i \rightarrow 0$ and $\beta_j \rightarrow 0$ for all values of α_i, α_j in $[0, 1]$.

4.3 The general participation result

We now show that our variance-compression results can be used to argue that the distribution of the shock expected by a player not participating in the redistribute agreement is a mean-preserving spread of the shock expected by agreement members.³¹ As a consequence, we can show that a sufficiently variance-averse player will prefer to join the agreement over staying outside.

³¹ See Rothschild & Stiglitz (1970) for the idea of mean-preserving spreads as a measure of risk.

In the simplest case of a n -member, redistributive mechanism (superscript rm), a symmetric agreement member's expected equilibrium payoff is

$$\mathbb{E}[u^{rm}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{rm})] = \int_{-\infty}^{+\infty} (\tilde{y}(e^*) + \varepsilon^{rm} - s_i m(n e^* - n f^*) - c_e(e^*) - c_f(f^*)) d\hat{F}(\varepsilon^{rm}) \quad (20)$$

in which ε^{rm} summarises the total sum of variances from the employed (contest) mechanism and the individual payoff shocks; we call the distribution of this compound variable $\hat{F}(0, \hat{\sigma}^2)$. Given an existing agreement with $n-1$ participants, the equilibrium payoff of a player $i \in \mathcal{N}$ who is free-riding (superscript fr) on the abatement efforts of the agreement is

$$\mathbb{E}[u^{fr}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{fr})] = \int_{-\infty}^{+\infty} \left(\tilde{y}(\tilde{e}) + \varepsilon^{fr} - s_i m(\tilde{e} + (n-1)e^* - \tilde{f} - (n-1)f^*) - c_e(\tilde{e}) - c_f(\tilde{f}) \right) dF(\varepsilon^{fr}) \quad (21)$$

in which $\tilde{\mathbf{e}}, \tilde{\mathbf{f}}$ are equal to $\mathbf{e}^*, \mathbf{f}^*$ with the free-rider's positions replaced by $\tilde{e}, \tilde{f}$. We show below that, under a sufficiently concave transformation v of the utility implied by (20) and (21), we can ascertain agreement participation, i.e., that $\mathbb{E}[v(u^{rm}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{rm}))] \geq \mathbb{E}[v(u^{fr}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{fr}))]$. Notice that, for the purpose of deriving a sufficient condition, we can ignore the influence of the individually suffered damage share $s_i m(n e^* - n f^*) < s_i m(\tilde{e} + (n-1)e^* - \tilde{f} - (n-1)f^*)$ which works in our favour.

Existing results for the case of $\varepsilon^{rm} = \varepsilon^{fr} \equiv 0$ show,³² that the typical case is $\mathbb{E}[u^{rm}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{rm})] \leq \mathbb{E}[u^{fr}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{fr})]$ which implies that no agreement is formed because

$$u^{rm}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{rm}) \Big|_{\varepsilon^{rm} \equiv 0} < u^{fr}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{fr}) \Big|_{\varepsilon^{fr} \equiv 0}. \quad (22)$$

In our stochastic environment, remark that ε^{fr} and ε^{rm} are two random variables which follow two specific—zero mean—distributions, $\varepsilon^{fr} \sim \mathcal{L}(0, \sigma^2)$ and $\varepsilon^{rm} \sim \mathcal{L}(0, \hat{\sigma}^2)$ with $\sigma^2 > \hat{\sigma}^2$, i.e., $\mathbb{V}[\varepsilon^{fr}] > \mathbb{V}[\varepsilon^{rm}]$. Indeed, in contrast to an isolated free rider whose income shocks are distributed as for independent individuals, agreement members can mutualise risk to some extent through pooling their resources. As shown in propositions 5 & 6 for the contest game and propositions 7 & 8 for general (riskless) redistributive mechanisms, the *ex-post* variance of the distribution of partly mutualised risk is lower than standalone income variance, even if we consider the additional equilibrium income variance created through a contest. Since the symmetric equilibrium solutions to (20) and (21) are invariant under increasing concave transformations, the equilibrium choice of effort does not change

³² Solid theoretical arguments against agreement participation in the deterministic case were derived, for instance, by Diamantoudi & Sartzetakis (2006) and Guesnerie & Tulken (2009).

if the concerned decision makers change their degree of risk aversion. Hence, there exists a function $v(\cdot)$, with $v'(\cdot) > 0, v''(\cdot) \leq 0$, which leads to $\mathbb{E}[v(u^{rm}(e^*, f^*; \varepsilon^{rm}))] \geq \mathbb{E}[v(u^{fr}(\tilde{e}, \tilde{f}; \varepsilon^{fr}))]$ for any positive difference $\mathbb{V}[\varepsilon^{fr}] - \mathbb{V}[\varepsilon^{rm}]$. Our main participation result then follows immediately.

Proposition 9. *For every positive difference of the equilibrium variances between the redistributive and the free-riding mechanisms, there is a family of concave functions v which provides a higher payoff to the redistributive mechanism.*

This result shows that, in our stochastic setup, there is a degree of risk aversion which leads to full participation in the symmetric redistribution agreement. An illustration of the intuition is attempted in figure 1.

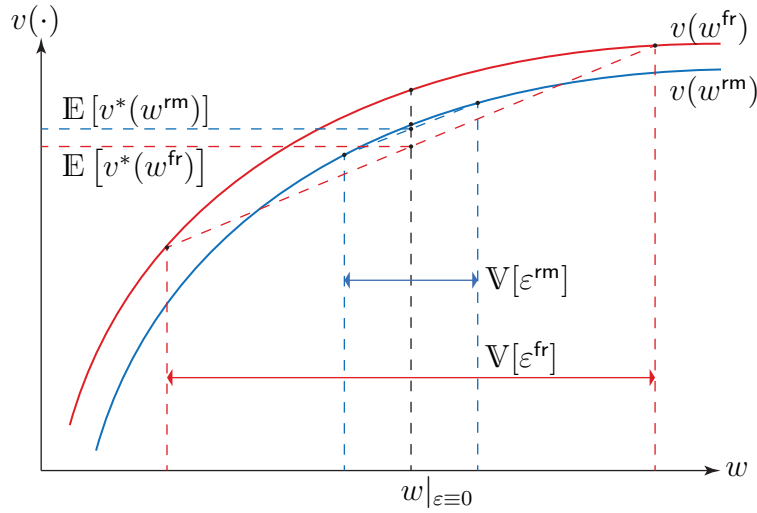


Figure 1: Variance compression leads to participation under sufficient variance aversion.

In this section, we present a redistributive agreement model in which an individual member's variance of income is compressed through joining the agreement. Under the assumptions we make, variance aversion implies risk aversion and, hence, risk averse players prefer less to more income variance. The variance compression is reached on the part of income which is pooled and redistributed among members: the risk sharing is an effect of the reduced risk of the pooled and redistributed income. We use this property of redistributive agreements in both a contest and a sharing model which implements efficient effort choices among nations facing multiple external effects. Since this balanced budget contest must redistribute wealth in order to implement efficient efforts—through awarding member states prizes—the efficient contest compresses the income risk of member states.

Although the model we present is highly stylised and our distributional assumptions are made

for reasons of modelling simplicity rather than realism, the insurance property can be derived as an entirely general feature of redistributive mechanisms. This insurance aspect is a formidable reason to join international agreements which, so far, seems to have been overlooked in the agreement formation literature. Since, moreover, our variance compression results are already obtained for small agreements (with a small number of participants), this paper presents a theoretical rationale to start an agreement among a handful of progressive nations, let them reap the benefits of risk sharing, and to only gradually enlarge the successful international agreement.

5 A simple symmetric example

To put the formal results of this paper into context, this section illustrates our main efficiency findings through example. We show in a simple setup that it is possible to reach the efficient allocation among symmetric players who consent to the agreement parameters. We continue to build on this example in the online appendix to illustrate further results. All examples share the same quadratic costs and square root production function to demonstrate the basic ideas. In this setup, a benevolent planner maximising the sum of social utility net of total cost (2) maximises

$$\max_{(e,f)} 2e^{1/2} - (2e - 2f)^2 - 2(e^2 + f^2) \Leftrightarrow \begin{cases} e^* \approx 0.2823, \\ f^* \approx 0.1882. \end{cases} \quad (23)$$

The corresponding individual problem (in the absence of an incentive mechanism) leads to inefficient provision of efforts because

$$\max_{(e_i, f_i)} e_i^{1/2} - s_i(e_i + e_j - f_i - f_j)^2 - (e_i^2 + f_i^2) \Leftrightarrow \begin{cases} e \approx 0.3029 > e^*, \\ f \approx 0.1514 < f^*, \end{cases} \quad (24)$$

for symmetric damage shares $s_1 = s_2 = 1/2$. Notice that, with respect to the efficient efforts, the combined externality and free-riding inherent in the problem imply that players both produce too much and abate too little.

For our incentive agreement we assume in the present example that the probability of winning the reduction award is given by the (generalised) Tullock success function specifying a player's probability of winning as a function of that player's effort over the total sum of efforts.³³ The prize pool which

³³ Under a Tullock contest success function, the contestant with the highest effort does not necessarily win the prize. Hence, the resulting ranking has occasionally been referred to as 'non-fully discriminatory,' 'non-deterministic,' 'noisy,' or 'fuzzy.' Our interpretation is that the ranking is inexact in the sense that the monitoring technology it is based on is not perfect. The Tullock (or Logit) form has been axiomatised by Skaperdas (1996) and follows naturally from micro-foundations à la Fu & Lu (2012) or Jia (2008).

we collect for incentive purposes is $P = (1 - \alpha)(e_i^{1/2} + e_j^{1/2})$. Then, an individual's problem under the incentive scheme is

$$\max_{(e_i, f_i)} \alpha e_i^{\frac{1}{2}} + \frac{f_i^r}{f_i^r + f_j^r} \beta P + \frac{f_j^r}{f_i^r + f_j^r} (1 - \beta) P - s_i (e_i + e_j - f_i - f_j)^2 - (e_i^2 + f_i^2) \quad (25)$$

for some exponent $r > 0$ specifying the precision with which the ranking selects the highest reduction effort nation among the set of competitors.³⁴ We interpret this exponent as the accuracy with which the agreement monitors the emissions reduction efforts of its members. Whenever we consider the Tullock example case in the following, we write the corresponding contract as $\langle \alpha, \beta; r \rangle$ instead of the general ranking based contract $\langle \alpha, \beta; p(\mathbf{f}) \rangle$.

Upon maximisation, this gives the two simultaneous first-order conditions

$$16e_i = 8f_i + \frac{1 + \alpha}{\sqrt{e_i}}, \quad 2e_i = 4f_i + \frac{\sqrt{e_i} r (\alpha - 1) (2\beta - 1)}{2f_i}. \quad (26)$$

We again consider the simplest case in which symmetric nations are identical (as we did before for the planner) and set $e = e_1 = e_2$, $f = f_1 = f_2$, with $s_i = 1/2$. We then force the resulting efforts in line with the efficient efforts by imposing $e = e^*$ and $f = f^*$ from (23) and solve (26) for the efficiency inducing design parameters $\langle \alpha, \beta; r \rangle$

$$\alpha^* = \frac{3}{5}, \quad \beta^* = \frac{1}{2} + \frac{1}{6r}. \quad (27)$$

As β^* depends on the precision of the monitoring technology r , the rewards scheme—and in particular the relative size of the prizes paid to the winner and loser given by β —can be designed as seen fit and compatible with the chosen monitoring technology.³⁵ The mechanism satisfies $\beta \in [\frac{1}{2}, 1]$ if $r \geq \frac{1}{3}$, implying that the losing nation never needs to pay more than the committed share $1 - \alpha$. Figure 2 shows that participating in the contest gives higher utility than staying out and free-riding on the other's effort. It confirms $(\alpha^*, \beta^*, e^*, f^*)$ as equilibrium in pure strategies with full participation (on an appropriately chosen plot-range outside of which utility is negative).

The economics of this example is simple: An increase in productive efforts e_i causes individual output $y(e_i)$ —and, hence, the prize pool P —as well as global pollution $m(\sum_h e_h - \sum_h f_h)$ to

³⁴ The particular monitoring technology is not very important as we generalise over the set of applicable success functions in online appendix A.2. What is important is that the success function incorporates enough randomness in its outcome. If the ranking is too precise (as is the case with the all-pay auction—which can be viewed as the $r = \infty$ limit-case of the Tullock function) then equilibria in pure strategies typically fail to exist. This would be problematic as our contest strives to implement the efficient pure effort choices.

³⁵ There are well-known existence issues with symmetric pure-strategy equilibrium with $r > 2$ in rent-seeking contests (see, e.g., Schweinzer & Segev (2012)) but, as shown in proposition 2, these do not apply with the same severity to our problem in which costs are convex and the prize pool is endogenous.

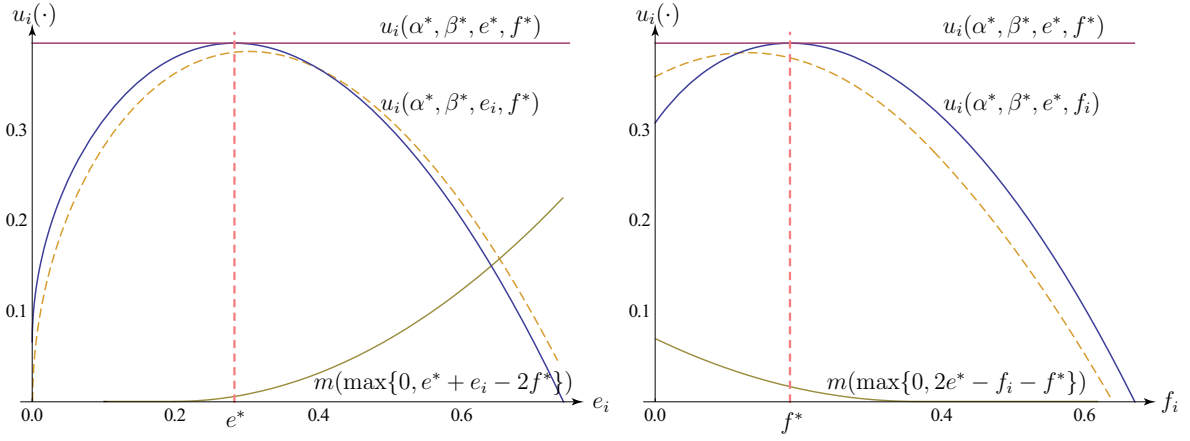


Figure 2: The top, horizontal line is the equilibrium utility from $(\alpha^*, \beta^*, e^*, f^*)$ implementing efficient efforts e^* and f^* . The curves below show the utility from unilaterally deviating in either effort dimension. Notice the positive utility from free-riding at zero efforts. The dashed curves give the (outside) utility from no agreement formation exhibiting both overproduction in e_i and underprovision of abatement f_i relative to the socially efficient levels.

rise. Of these, the player retains shares α and s_i , respectively. An increase in abatement efforts f_i enlarges the player's chance to win the prize share β in the reduction contest (while decreasing the competitors' chances) and simultaneously decreases global pollution. Trading off α against β allows us to fine-tune efforts to their efficient levels. A simple quantification based on 2011 global GDP in online appendix D gives a feeling for the magnitudes of redistribution implied by this two players example. A whole sequence of examples which extend this main example and illustrate the paper's further results is provided in online appendix D.

6 Concluding remarks

This paper gives theoretical reasons for independent nations which are sufficiently averse to income shocks to agree on a simple, redistributive contest, organised among themselves, which can implement both efficient productive and pollution abatement efforts. Our formal results rely critically on assumptions about the curvature and separability of the involved cost and production functions. As customary in the IEA literature we assume that the abatement costs are increasing and convex (see, e.g., Barrett 2006, Diamantoudi & Sartzetakis 2006 and Carraro et al. 2009). We make similar assumptions on the damage function which are criticised, among others, by Weitzman (2010) but convexity seems nonetheless the most commonly used form (Revesz et al., 2014). Even if all these technical conditions turn out to be innocuous, the desirable characterisation of many properties

of this mechanism must be left for future work: Which share of global (per capita) GDP would have to be redistributed—in reality—to the country with the highest emissions reduction in order to implement our results? Is the resulting wealth redistribution one we would like to see? Can the mechanism's design parameters be effectively negotiated? Can international (emissions certificate) trade be successfully incorporated into the model? Answers to all these questions have significant policy implications, are to a large extent empirical and are at least partly determined by politics. At any rate we do not feel qualified to answer these questions now.

There is, however, a set of immediate challenges to the mechanism we propose which we can respond to now and would like to address in the remainder of this concluding discussion. *i)* Measurement of output. At the national level, y corresponds to GDP. But GDP measurement relies on approximation, and GDP estimates are often revised. If we are seriously contemplating large international cash transfers that depend on national output figures, the accuracy, and manipulability, of those measures is a concern. While it is not sufficient for the measure of y to be 'right on average' in order to achieve efficiency, it is also true that our measure of efficiency (welfare maximisation) relies on the same measurement imperfections. So our mechanism is as good as a *complete information* mechanism can be in this setup. Obviously, introducing private information would improve the realism of our setup greatly—but since we do not have a model implementing efficiency and ensuring participation even under full information, we are reluctant to attempt a solution of the incomplete information case directly.

ii) Non-manipulability of the 'abatement effort monitoring device.' Similarly, the manipulability of any monitoring device must be an issue for our mechanism. It is remarkable, however, that monitoring of abatement efforts f as required by our model does not need to be perfect—on the contrary, the precision of the detector is a design element of the agreement we propose. Imperfect discrimination is one of the main features of the contest technology we employ.

iii) Commitment to share-of-output payments. By becoming an agreement member, a country pledges a certain share of national output. Reneging on this pledge is a political choice which we view as equivalent to leaving the agreement. The paper discusses several scenarios in which joining the agreement is rational: all of these are also effective in avoiding agreement desertion in the form

of withdrawal of pledged resources.³⁶

iv) Separation between productive and abatement efforts. As we model efforts as either productive or reductive, a technology in our model cannot, apparently, be both productive *and* pollution reductive. This is only a model simplification: adding another concave production function based on abatement efforts to the maximisation problem would make our sufficient existence condition *easier* to satisfy.

v) The treatment of countries as single, profit-maximising decision makers. While this is a standard modelling assumption, the micro-politics of decision making on production (or abatement) levels may well be much more challenging than suggested by our simple model. Who pays or receives the marginal benefit of transfers through these contests? Who actually owns the output and therefore funds the prize pool? Although our model cannot address these questions in its present form, our main ideas could be equally well applied on the state or municipal levels where the micro-actors would be much easier to identify. In a similar vein, another immediate application possibility is to ‘smaller’ abatement competitions at separate industry levels of the participating countries.³⁷

vi) Unrealistically large transfers. While we present the incentives in our mechanism in terms of winning probabilities, we would like to stress that an equivalent interpretation in terms of actual effort-dependent shares of the prize pot is possible without changing our efficiency results. The obvious advantage of such a mechanism is that, in symmetric pure-strategy equilibrium, all payments cancel each other out and no net-transfers are necessary. On the downside are contractibility issues and that the statistical (cost) advantages of requiring only ordinal information to determine a ‘winner’ are lost.

We neither belittle nor shrug off any of these important problems an actual agreement needs to solve. To a large extent, however, we feel that *any* mechanism attempting to solve the emissions problem will have to face a variant of these problems. The present paper attempts to name and discuss these challenges—and provides first results showing that a mechanism along the lines we indicate can *in theory* correct nations’ combined incentives to emit too much while abating too little.

³⁶ We would also like to point out that there are several existing international agreements and unions among (formerly) independent states which redistribute pledged resources in a way not too distant from our model. The main examples of comparable redistribution are (very stylised versions of) the European Union and, despite many instances of laggardly payment, the United Nations.

³⁷ We are grateful to an anonymous referee for pointing out to us that unpacking welfare beyond national aggregates has nontrivial consequences for decision-making. The link between government policy and special interests has been analysed theoretically for abatement decisions in IEAs by Dietz et al. (2012) as well as empirically with regards to agricultural trade by Gawande & Hoekman (2006).

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Agreeing on efficient emissions reduction: *Online appendix**

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This online appendix contains proofs, further details, and examples which could—due to space restrictions—not be included in the main manuscript. We identify equation (n) in the main text using the reference M:(n); all other references are made to this appendix.

Appendix A: Extensions and robustness

A.1 Enforcing participation in the deterministic output model

The purpose of this section is to show that a simple way to deter free-riding of individual nations even in the deterministic output version of our model is to grant most favoured ‘green’ trading terms only to participating nations. Similarly, environmental certification conditional on treaty commitment can be a powerful complementary tool to enforce participation. We would like to point out prominently, however, that participation in the deterministic model version is generally not renegotiation proof. This is the reason why we introduce the more realistic, stochastic output version of our model in section 4 of the main text which supports agreement formation through an insurance motive.

The expansion of equilibrium abatement efforts from their no agreement level f^0 to the level within the agreement f^* creates ‘green’ products; the higher abatement efforts, the greener is productive output. The idea is to label free-riding countries’ products and thereby creating (political) incentives to respect commitments to the IEA. Consequently, firms’ lobbying against environmental regulation may result in that country’s desertion followed by the IEA labelling its goods. We thus propose a negative label which signals a product lacking the ‘green’ environmental standards enforced by the IEA.¹

This idea can be formalised in our setup by decreasing the value of a deserting country’s output. Loosely speaking, we require that—once certified as environmentally unfriendly—a consumer’s

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¹ There are many green labelling examples: UK supermarket chain Tesco has recently introduced a promotional campaign on carbon labels. US Walmart and French Casino have similar ambitions. Examples of negative labelling campaigns are the mandatory GMO labelling implemented in Europe, and “we’re Greenpeace, and we want a fresh green Apple” targeted at US computer maker Apple. Grankvist et al. (2004) argue that negative labelling may have a higher consumption impact than positive labelling. Engel (2004) underlines the necessity to inform consumers, especially when a firm is found cheating on its environmental claims.

willingness to pay for labelled products decreases.² Therefore, the revenue generated from the production of labelled products sinks as these products suffer a decrease in price of $x(\mathbf{f}) \in [0, 1]$. This fraction corresponds to the deserter's deviation from agreed abatement levels. Denoting the outside equilibrium efforts of a single deserter by (e^d, f^d) , desertion utility is

$$u_i^d(e_d, f_d) = \underbrace{\left(\frac{f^* - f_i^0}{f^*} \right)}_{=x \in [0,1]} y(e_i^d) - s_i m(e_i^d + (n-1)e^* - f_i^d - (n-1)f^*) - c_e(e_i^d) - c_f(f_i^d) \quad (1)$$

for $i \in \mathcal{N}$ and $n > 2$ (since there must be at least two players left in the agreement after i deserts). A sufficiently large fraction x will successfully deter free-riding on reductive efforts and can be seen as alternative to the blunt global threat represented by contract C' .³

A further step in this direction is the formation of an exclusive trade agreement. If a deserter can be excluded from the fraction of trade corresponding to the necessary abatement investments within the agreement, then individual desertion can, again, be discouraged. As above, green production—generated by the expanded abatement effort—is traded among countries and produces wealth. Our model does not take into account the international trading aspects of production and thus there is no direct way of measuring the involved consequences to individual wealth.⁴ A simple (ad-hoc) way of nevertheless capturing the idea of enforcement through an exclusive trading agreement is to restrict trade on (and therefore capitalising on) the fraction of productive output corresponding to the reductive investments within the treaty to agreement members. Then player i 's desertion utility in (1) can be reduced by using

$$x(\mathbf{f}) = \left(1 - \frac{f^* - f_i^0}{e^*} \right) \quad (2)$$

in which reductive effort f_i^0 is the equilibrium abatement level without agreement. As indicated above, to some extent this choice is arbitrary.⁵ The intuition for (2) is that an agreement defector j can free-ride on the reductive efforts of agreement members (through a cleaner environment) but is punished by restricted access to the agreement market consisting of the tradables $\sum_{h \neq j} y(e_h)$.

Returning to the simple quadratic cost, square-root production example of section 5 in the main text, this implies that it is individually rational to participate in the agreement if

$$u_i(e^*, f^*) \geq x(\mathbf{f}) e_i^{\frac{1}{2}} - \frac{1}{n} (e_i + (n-1)e^* - f_i - (n-1)f^*)^2 - e_i^2 - f_i^2. \quad (3)$$

Solving the deserter's maximisation problem (on the rhs) for the exclusive trade agreement under (2) leads to the first-order conditions

$$2f^* + \frac{e^* - f^* + f_i^0}{2e^* \sqrt{e_i^d}} = \frac{2(e_i^d + f^* - f_i^d + e^*(n-1) + e_i^d n)}{n}, \quad f_i^d = \frac{e_i^d + (e^* - f^*)(n-1)}{n+1} \quad (4)$$

² Models on certification and standard settings have been studied intensely; see for instance Lerner & Tirole (2006) or Harbaugh et al. (2011)

³ To some extent, the particular choice of $x(\mathbf{f})$ is arbitrary. Any function of abatement efforts implementing sufficient deterrence (such as the function used for the exclusive trade example below) could be used instead.

⁴ Modelling both these aspects formally is possible and certainly provides grounds for future research.

⁵ If monitoring of the deserter's abatement efforts is good, then the actual level f_i^d could be used. (In the present example setup, this leads to a rather unintuitive corner solution.) The idea, however, seems important for dealing with competing abatement agreements: As long as they are effective, there is no reason to punish them.

in which (e^*, f^*) are the equilibrium effort levels provided inside the agreement. Plotting the utilities from the desertion efforts (e_i^d, f_i^d) solving above first-order conditions results in a graph similar to figure 2 in the main text for the case of $n = 3$ showing that deviations are not profitable. Thus, the exclusive trade agreement ensures participation. (The analysis is nearly identical for the labelling setup discussed above and therefore not replicated.) As the severity of punishment $(1 - x(f))$ in (2) is given by

$$\frac{f^* - f_i^0}{e^*} = \frac{6 - 3^{1/3}(-2 - 4n)^{2/3}(-1 - n)^{1/3}}{6(1 + n)} \quad (5)$$

which is increasing in n , the punishment gets more severe for larger n and participation is easier to obtain in the general case. (The limit as $n \rightarrow \infty$ equals $2^{1/3}/3^{2/3} \approx .606$.)

A.2 The choice of ranking technology

Consider a n -player extension of the problem of section 5 in the main text with prize structure $(\beta, \frac{1-\beta}{n-1}, \dots, \frac{1-\beta}{n-1})$. The present example shows that efficiency can also be obtained in proposition 2 for a 'difference-form' success function. The specific properties of the generalised Tullock success function which we use in the remainder of the paper are therefore not crucial to our results. Difference-form success functions have been widely used in the literature, for instance by Che & Gale (2000), but suffer from the lack of a generally accepted, simple extension to more than two players. We define player i 's probability of winning as

$$p_i(\Delta) = \frac{\exp^{\Delta_i^r}}{\sum_{j=1}^n \exp^{\Delta_j^r}}, \text{ where } \Delta = (\Delta_1, \dots, \Delta_n), \Delta_i = f_i - \frac{\sum_{j \neq i} f_j}{n-1}, \text{ and } r > 0. \quad (6)$$

Setting $P = (1 - \alpha)(e_i^y + (n-1)e_j^y)$, $y \in (0, 1)$, $m, b > 1$ and all $j \neq i$ equal, player i 's individual problem is to

$$\max_{(e_i, f_i)} \alpha e_i^y + p_i(\Delta) \beta P + (1 - p_i(\Delta)) \frac{1 - \beta}{n-1} P - s_i(e_i + (n-1)e_j - f_i - (n-1)f_j)^m - (e_i^b + f_i^b)$$

which, in symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$ gives for any $p_i(\Delta)$

$$\alpha = \frac{e^{-y} ((e - f)(be^b n - e^y y) + em((e - f)n)^m s_i)}{(e - f)(n-1)y},$$

$$\beta = \frac{e^{-y} (-b(e - f)f^b(n-1)n + f(m(n-1)((e - f)n)^m s_i + e^y(e - f)n^2(\alpha - 1)p'_i(0)))}{(e - f)fn^3(\alpha - 1)p'(0)}$$

in which $\Delta = 0$ is the equilibrium vector of deviations.

Plugging in the efficient efforts from M:(23), employing (6), and returning to the example setup from section 5 (i.e., $n = 2$, $y = 1/2$, $b = m = 2$, and $s_i = 1/2$), this results in a very similar efficient mechanism as under the Tullock success function

$$\alpha^* = \frac{3}{5}, \beta^* = \frac{r + (5/6)^{2/3}}{2r} \quad (7)$$

in which $\beta^* \in (.5, 1]$ is ensured for $r \geq (5/6)^{2/3} \approx 0.89$. A picture nearly identical to figure 2 confirms, for instance, $\langle \alpha^*, \beta^*, r = 2 \rangle$ as equilibrium. The precise form of ranking technology employed is thus immaterial to our results.

Appendix B: Proofs

Proof of proposition 1. Efficient efforts are extending M:(2) as the pair (e^*, f^*) solving

$$y'(e) = m'(ne - nf) + c_e(e, f), \quad m'(ne - nf) = c_f(e, f). \quad (8)$$

Let $P = (1 - \alpha) \sum_{h=1}^n y(e_h)$. Since we are only interested in deviations from symmetric equilibrium, we set $e_j = e_{-i}$. Rewriting M:(8) for the 2-prize structure $(\beta^1, \frac{1-\beta^1}{n-1}, \dots, \frac{1-\beta^1}{n-1})$ results in

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \sum_{h=2}^n \frac{1 - \beta^1}{n - 1} p_i^h(\mathbf{f})P - s_i m(e_i + (n - 1)e_j - f_i - (n - 1)f_j) - c_e(e_i) - c_f(f_i)$$

which simplifies to

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \frac{1 - \beta^1}{n - 1} (1 - p_i^1(\mathbf{f}))P - s_i m(e_i + (n - 1)e_j - f_i - (n - 1)f_j) - c_e(e_i) - c_f(f_i).$$

The symmetric $e = e_i = e_j$, $f = f_i = f_j$, first-order conditions for this problem are

$$\begin{aligned} c'_e(e) + s_i m'(ne - nf) &= \frac{1 - \beta^1 + \alpha(n + \beta^1 - 2) + (1 - \alpha)(n\beta^1 - 1)}{n - 1} p(f) y'(e) \\ c'_f(f) &= s_i m'((e - f)n) + \frac{n(1 - \alpha)(n\beta^1 - 1)}{n - 1} p'(f) y(e). \end{aligned} \quad (9)$$

Plugging in (8) and imposing $s_i = 1/n$, one obtains

$$\alpha^* = 1 - \frac{y'(e^*) - c'_e(e^*)}{y'(e^*)} \text{ and } \beta^* = \frac{1}{n} + \frac{(n - 1)^2 y'(e^*)}{n^3 y(e^*) p'(f^*)} \quad (10)$$

which can be achieved for any exogenously given ranking technology $p(\mathbf{f}^*)$. $\square$

Proof of proposition 2. Since under our assumptions M:(8) is fully separable we can split the problem into two independent problems along the respective effort dimensions. Setting $P = (1 - \alpha) \sum_{h=1}^n y(e_h)$, the two separate problems are

$$\begin{aligned} \alpha y(e_i) + \beta^1 p_i^1(\mathbf{f}^*)P + \frac{1 - \beta^1}{n - 1} (1 - p_i^1(\mathbf{f}^*))P - s_i m(e_i + (n - 1)e^* - n f^*) - c_e(e_i) - c_f(f^*), \\ \alpha y(e_i^*) + \beta^1 p_i^1(\mathbf{f})P + \frac{1 - \beta^1}{n - 1} (1 - p_i^1(\mathbf{f}))P - s_i m(ne^* - f_i - (n - 1)f^*) - c_e(e^*) - c_f(f_i). \end{aligned}$$

1) We show that exerting productive effort $e_i = e^*$ gives a global maximum. As players are symmetric and we are looking for a profitable deviation from the efficient level we set $\mathbf{f}^* = (f_1 = f^*, \dots, f_n = f^*)$ implying that the probability of winning is $p_i^1(\mathbf{f}^*) = 1/n$. Thus the problem simplifies to

$$\alpha y(e_i) + \frac{1}{n} P - s_i m(e_i + (n - 1)e^* - (n)f^*) - c_e(e_i) - c_f(f^*) \quad (11)$$

giving the first-order condition for productive effort e_i as⁶

$$\underbrace{y'(e_i) \left(\alpha + \frac{1}{n} (1 - \alpha) \right)}_{\searrow} = \underbrace{s_i m'(\max\{0, e_i + (n - 1)e_j^* - (n)f^*\})}_{\nearrow} + \underbrace{c'_{e_i}(e_i)}_{\nearrow}.$$

⁶ It is routine to verify that both first-order conditions identify a maximum.

Notice that output is strictly increasing in e_i and is strictly concave. Thus $y''(e_i) < 0$ and $y'(e_i)$ is decreasing. Both cost functions are increasing and convex, therefore $s_i m''(\cdot) + c''(e_i) > 0$ and the rhs is increasing. As $y'(0) > s_i m'(\max\{0, (n-1)e^* - nf^*\}) + c'(0)$,⁷ this confirms single crossing of rhs and lhs and ensures the existence of an equilibrium.

2) We now demonstrate global optimality of $f_i = f^*$. Assuming efficient productive effort provision, the first-order condition for reductive effort is

$$\underbrace{ny(e^*)(1-\alpha)(\beta n-1)p'(f_i, f^*)}_{=B} = \underbrace{c'_{f_i}(f_i)}_{=C \nearrow} - \underbrace{s_i m'(\max\{0, ne^* - (n-1)f^* - f_i\})}_{=A \searrow}. \quad (12)$$

Notice that the rhs is strictly increasing as we know that, with respect to f_i , $s_i m''(\cdot) \leq 0$ and thus that A is decreasing and the cost function is convex. Without further assumptions on the monitoring technology $p(\cdot)$ we cannot sign the slope of B . Notice, however, that increasing the slope of the (convex) cost function $c'_{f_i}(f_i)$ sufficiently guarantees single crossing and thus a unique maximum whatever the precise specification of $p(\cdot)$.

3) We now show that (12) identifies a global maximum for the Tullock success function.⁸ Again, $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) > 0$ for $f_i = 0$ while $p'(f_i, f^*) = 0$ and thus the lhs of (12) is zero at $f_i = 0$ while the rhs is negative. Single crossing is immediate for the case of $r \in (0, 1]$ as B is (weakly) decreasing. In the general case of

$$p_i(\mathbf{f}) = \frac{f_i^r}{\sum_{j=1}^n f_j^r}, \quad r > 1, \quad (13)$$

the function B has a single critical point and is decreasing when $f_i \geq f^* \left(\frac{(n-1)(r-1)}{r+1} \right)^{1/r}$.

To get single crossing if the two curves are increasing we need to ensure either strict concavity or convexity for the lhs and strict convexity for the rhs and prove that if $f_i = 0$, lhs is larger than the rhs. As we have not specified anything about our functions regarding the third derivative we illustrate this point using the specific $c'_{f_i}(f_i) = bf^{b-1}$ and $s_i m(\max\{0, ne^* - (n-1)f^* - f_i\}) = s_i (\max\{0, ne^* - (n-1)f^* - f_i\})^b$. We also set $s_i = \frac{1}{n}$. We find that both curves have an inflection point, thus we need to find a condition to ensure single crossing.

We first show that the rhs starts out negative and eventually becomes positive as for $f_i = 0$ we have $C - A = -s_i m'(\max\{0, ne^* - (n-1)f^*\}) < 0$. Therefore, as long as the lhs is positive and the rhs negative, the two curves cannot cross. We find that $C - A < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-1}} + 1}$ because

$$C - A = f_i^{b-1}b - \frac{\overbrace{(ne^* - (n-1)f^* - f_i)}^{=2f^*}}{n}^{b-1}b = 0 \Leftrightarrow (2f^* - f_i)^{b-1} = nf_i^{b-1}. \quad (14)$$

Moreover, for the rhs, the inflection point occurs when the curve is negative, and it is first concave and then convex. Thus we can conclude that when the curve is above zero, it is strictly convex.

⁷ Since output is concave and the sum of cost functions is convex in e_i , the above inequality holds.

⁸ A nearly identical argument can be made for any other ratio-based success function. In that more general case, however, we cannot derive an explicit existence threshold.

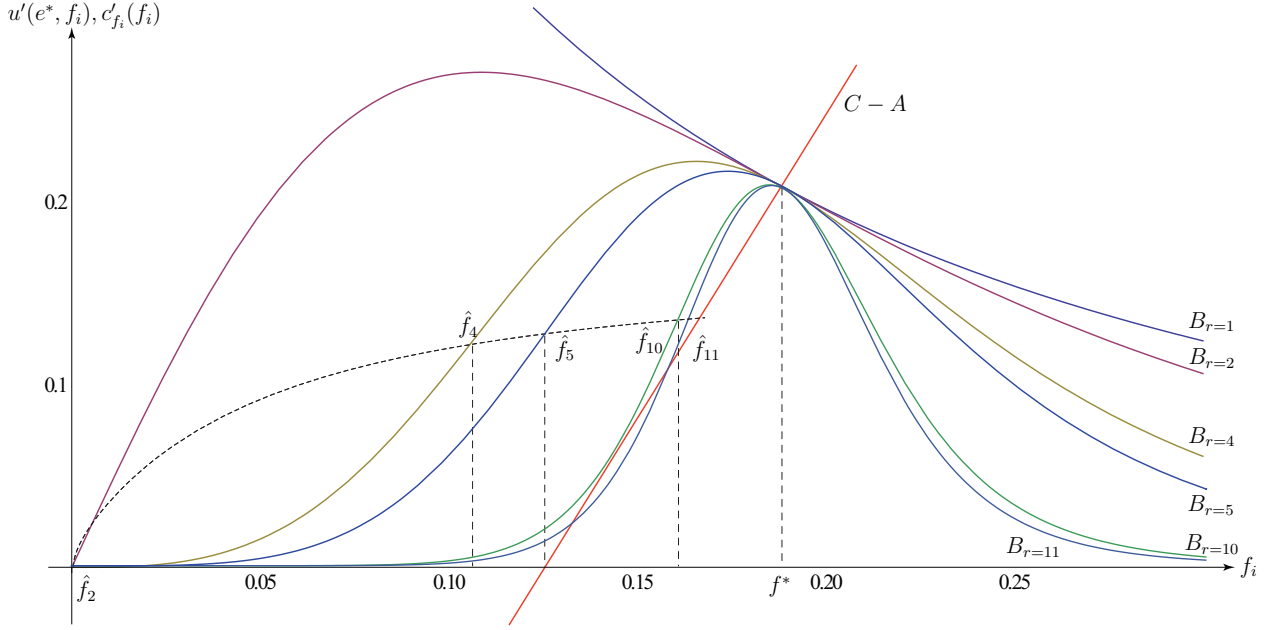


Figure 1: Single crossing in equation (12) ensures a unique global maximum at $f_i = f^*$ for the example setup of section 5. The dotted line gives the location of inflection points $\hat{f}$ for different r .

We find that $(C - A)'' < 0$ for $f_i < f^* \frac{2}{n^{\frac{1}{b-3}} + 1}$ and $f_i < f^* \frac{2}{n^{\frac{1}{b-3}} + 1} < f^* \frac{2}{n^{\frac{1}{b-1}} + 1}$ because⁹

$$\begin{aligned} (C - A)'' &= f_i^{b-3}(b-2)(b-1)b - \frac{(2f^* - f_i)^{b-3}(b-2)(b-1)b}{n} = 0 \Leftrightarrow (2f^* - f_i)^{b-3} = n f_i^{b-3}, \\ &\Leftrightarrow f^* \frac{2}{n^{\frac{1}{b-1}} + 1} - f^* \frac{2}{n^{\frac{1}{b-3}} + 1} = 2 \frac{f^* \left(n^{\frac{1}{b-3}} - n^{\frac{1}{b-1}} \right)}{\left(n^{\frac{1}{b-1}} + 1 \right) \left(n^{\frac{1}{b-3}} + 1 \right)} \geq 0. \end{aligned}$$

We conclude that the rhs is strictly increasing and convex when it is positive.

For the lhs, there are two inflection points: one in the increasing part and the other in the decreasing part. In the increasing part we find a condition which implies that the inflection occurs if the rhs is negative.¹⁰ A sufficient condition for single-peakedness is therefore that

$$\frac{2^r (f^*)^r}{\left(n^{\frac{1}{b-1}} + 1 \right)^r} \geq \underbrace{\frac{(n-1) \left(2(f^*)^r (r^2 - 1) - \sqrt{3} \sqrt{(f^*)^{2r} r^2 (r^2 - 1)} \right)}{2 + 3r + r^2}}_{=: \hat{f}}. \quad (15)$$

Thus if the rhs of (12) is positive, it is also strictly convex. If (15) is respected, the lhs is strictly concave or convex. Notice also that at the inflection point, the rhs is positive and the lhs is negative and therefore the lhs is larger than the rhs. The geometric intuition of (15) is shown in figure 1 for the setup of the example section 5 in the main text. The figure shows a family of curves B for $r \in \{1, 2, 4, 10, 11\}$ with inflection points labelled $\hat{f}_2$, $\hat{f}_4$, $\hat{f}_{10}$, and $\hat{f}_{11}$, respectively. Condition (15) is fulfilled as long as the red cost curve $C - A$ is negative at the respective inflection point. This is true for $r = \{2, 4\}$ but (15) fails for higher r . Uniqueness, however, is only lost for $r > 10$. $\square$

⁹ This is true for any $b \geq 3$.

¹⁰ The inflection point in the decreasing part does not matter. As long as one curve is increasing and the other is decreasing they can only cross once.

Proof of proposition 3. Player i 's equilibrium participation utility for $P = (1 - \alpha)ny(e^*)$ under the efficiency-inducing contract $C = \langle \alpha^*, \beta^*; p^*(\mathbf{f}) \rangle$ defined in (8) with prizes $(\beta, \frac{1-\beta}{n-1}, \dots, \frac{1-\beta}{n-1})$ is

$$\begin{aligned} u_i(e^*, f^*) &= \alpha^* y(e^*) + \frac{1}{n} \beta^* P + (n-1) \frac{1}{n} \frac{1-\beta^*}{n-1} P - s_i m(ne^* - nf^*) - c_e(e^*) - c_f(f^*) \\ &= \alpha y(e^*) + \frac{1}{n} (1 - \alpha^*) ny(e^*) - s_i m(ne^* - nf^*) - c_e(e^*) - c_f(f^*) \\ &= y(e^*) - s_i m(ne^* - nf^*) - c_e(e^*) - c_f(f^*). \end{aligned} \quad (16)$$

Now consider the contract $C' = \langle \alpha', \beta'; p' \rangle$. Since second-stage efforts $e(\alpha', \beta', p'), f(\alpha', \beta', p')$ are continuous in α' , it is sufficient to consider the extreme case of $\alpha' = 1$ which implements no contest at all. A deserter's utility when subjected to C' can therefore be driven down to¹¹

$$u_i^s(e_i^s, f_i^s) = y(e_i^s) - s_i m(e_i^s + (n-1)e^a - f_i^s - (n-1)f^a) - c_e(e_i^s) - c_f(f_i^s) \quad (17)$$

in which e^a and f^a are the equilibrium inside agreement efforts prescribed by C' . For $\alpha' = 1$, these equal the equilibrium efforts without agreement e_i^s, f_i^s . Hence (16) and (17) are identical but implement different efforts. Since both $e^s > e^*$ and $f^s < f^*$, it is therefore individually rational to join the agreement implementing C if the alternative is C' because the cost differential

$$s_i(m(ne^s - nf^s) - m(ne^* - nf^*)) \quad (18)$$

is increasing in n and convex. As the efficient allocation is welfare maximising, this outweighs any productivity gains from free-riding $y(e^s) - y(e^*) + c_e(e^*) + c_f(f^*) - c_e(e^s) - c_f(f^s)$. Thus, every player finds it individually rational to join the reductive contest if threatened by alternative C' . $\square$

Proof of proposition 4. From (12), we know that

$$\alpha_i = \frac{y'_i(e_i)(1 - H_i) - (1 - s_i)m'(G)}{y'_i(e_i)(1 - H_i)}, \text{ in which } H_i = \beta_i p_i^1(\mathbf{f}) + \frac{\sum_{j \neq i} (1 - \beta_j) p_j^1(\mathbf{f})}{(n-1)} \quad (19)$$

$$\beta_i = \frac{(n-1)((1 - s_i)m'(G)) - \sum_{j \neq i} (1 - \beta_j) p_{j(f_i)}^1(\mathbf{f}) P}{p_{i(f_i)}^1(\mathbf{f})(n-1)P}. \quad (20)$$

Equation (20) can be rewritten as

$$p_{i(f_i)}^1(\mathbf{f}) \beta_i = \frac{(1 - s_i)m'(G)}{P} - \sum_{j \neq i} p_{j(f_i)}^1(\mathbf{f}) \frac{1 - \beta_j}{n-1}. \quad (21)$$

Integrating over f_i , it follows that

$$p_i^1(\mathbf{f}) \beta_i = \frac{(1 - s_i)m(G)}{P} - \sum_{j \neq i} p_j^1(\mathbf{f}) \frac{1 - \beta_j}{n-1} \quad (22)$$

as $m'(G) = c'_{f_i}(f_i^*)$. Equation (22) in H_i leads to $1 - H_i = \frac{P - (1 - s_i)m'(G)}{P}$ which we insert into (19) to obtain

$$\alpha_i = 1 - \frac{(1 - s_i)m'(G)P}{y'_i(e_i)[P - (1 - s_i)m(G)]} \quad \forall i. \quad (23)$$

¹¹ Notice that the latter formulation requires $n > 2$ as the contest can only produce incentives if at least two players participate in the contest.

As $P = \sum_{j=1}^n y_j(e_j)(1 - \alpha_j)$, $(\alpha_1, \dots, \alpha_n)$ are described by a system of n equations independent of $(\beta_1, \dots, \beta_n)$ given by

$$\begin{aligned} & (1 - \alpha_i)^2 y'_i(e_i) y_i(e_i) + (1 - \alpha_i) y'_i(e_i) \sum_{j \neq i} y_j(e_j)(1 - \alpha_j) \\ & - (1 - s_i)(1 - \alpha_i) m(G) y'_i(e_i) + (1 - s_i) m'(G) \sum_{j=1}^n y_j(e_j)(1 - \alpha_j) = 0. \end{aligned} \quad (24)$$

From a fixed point argument such as, for instance, Brouwer's theorem, the vector $(\alpha_1, \dots, \alpha_n)$ exists. Then (20) leads to a system of linear equations and $(\beta_1, \dots, \beta_n)$ exists as well. $\square$

Proof of proposition 5. The variance of the pooled, n -player redistributed resource is

$$\begin{aligned} & \mathbb{V} \left[a \frac{y(e_1, \varepsilon_1) + \dots + y(e_n, \varepsilon_n)}{n} + b y(e_i, \varepsilon_i) \right] \\ &= a^2 \left(\frac{1}{n^2} \mathbb{V}(\varepsilon_1) + \dots + \frac{1}{n^2} \mathbb{V}(\varepsilon_n) \right) + b^2 \mathbb{V}(\varepsilon_i) + 2 \frac{a}{n} b \text{Cov}(\varepsilon_i, \varepsilon_i) \\ &= a^2 \frac{n}{n^2} \sigma^2 + b^2 \sigma^2 + \frac{2}{n} a b \sigma^2 \\ &= \sigma^2 \frac{a^2 + 2ab + nb^2}{n} \end{aligned} \quad (25)$$

for arbitrary constants a, b and any player $i \in \mathcal{N}$. Individual equilibrium contest payoff is the compound lottery

$$L = \begin{cases} \frac{1}{n} : & \beta^* \sum_{j=1}^n (1 - \alpha^*) y(e^*, \varepsilon_j) + \alpha^* y(e^*, \varepsilon_i) \\ 1 - \frac{1}{n} : & \frac{1 - \beta^*}{n - 1} \sum_{j=1}^n (1 - \alpha^*) y(e^*, \varepsilon_j) + \alpha^* y(e^*, \varepsilon_i). \end{cases} \quad (26)$$

Hence, the parameters a and b in (25) are

$$a = \beta^*(1 - \alpha^*) + (n - 1) \frac{1 - \beta^*}{n - 1} (1 - \alpha^*) = 1 - \alpha^*, \quad b = \alpha^* \quad (27)$$

which, when plugged into the last expression in (25), imply that the variance from the pooled output shocks alone is

$$\mathbb{V}_\varepsilon(\mathbb{E}(L|\varepsilon)) = \sigma^2 \frac{1 + (\alpha^*)^2(n - 1)}{n} < \sigma^2. \quad (28)$$

To this variance stemming from the individual shocks ε_i alone we have to add the variance introduced through the prize structure of the contest itself. The expectation of the contest payoff is

$$\mathbb{E}[L|\varepsilon] = \frac{1}{n} (1 - \alpha^*) \beta^* n y(e^*, \varepsilon) + \left(1 - \frac{1}{n} \right) \frac{(1 - \beta^*)}{n - 1} (1 - \alpha^*) n y(e^*, \varepsilon) + \alpha^* y(e^*, \varepsilon) = y(e^*, \varepsilon) \quad (29)$$

and, hence, $\mathbb{E}[L|\varepsilon] = y(e, \varepsilon)$ in which we drop the subscript on the individual shock because they are i.i.d.; the contest variance is therefore

$$\begin{aligned} \mathbb{V}(L|\varepsilon) &= \frac{1}{n} ((1 - \alpha^*) \beta^* n y(e, \varepsilon) + \alpha^* y(e, \varepsilon) - \mathbb{E}[L|\varepsilon])^2 + \\ & \quad \frac{1}{n} ((1 - \alpha^*) (1 - \beta^*) n y(e, \varepsilon) + \alpha^* y(e, \varepsilon) - \mathbb{E}[L|\varepsilon])^2 \\ &= \frac{(1 - \alpha^*)^2 (n((2(\beta^* - 1)\beta^* + 1)n - 2) + 2)}{n} \mathbb{E}[y(e, \varepsilon)^2]. \end{aligned} \quad (30)$$

Pulling (30) together with (28) gives, by the law of total variance $\mathbb{V}[L] = \mathbb{E}[\mathbb{V}[L|\varepsilon]] + \mathbb{V}[\mathbb{E}[L|\varepsilon]]$, the agreement member's payoff variance

$$\frac{\sigma^2((\alpha^*)^2(n-1)+1)}{n} + \mathbb{E}[y(e, \varepsilon)^2] \frac{(\alpha^* - 1)^2(n((2(\beta^* - 1)\beta^* + 1)n - 2) + 2)}{n} \quad (31)$$

which we need to be smaller than standalone income variance σ^2 . This condition holds if¹²

$$\mathbb{E}[y(e, \varepsilon)^2] \frac{(1 - \alpha^*)(n((2(\beta^* - 1)\beta^* + 1)n - 2) + 2)}{(\alpha^* + 1)(n - 1)} \leq \sigma^2. \quad (33)$$

Into this general threshold expression, we now substitute the equilibrium parameters of our standard square-root production $y = \sqrt{e}$ and quadratic costs example for n players

$$\begin{aligned} e^* &= \frac{\left(\frac{n+1}{2n+1}\right)^{2/3}}{2\sqrt[3]{2}}, & f^* &= \frac{n}{2\sqrt[3]{2}\sqrt[3]{(n+1)(2n+1)^2}}, \\ \alpha^* &= \frac{4\sqrt{e^*n}(2e^* - f^*) - 1}{n - 1}, & \beta^* &= \frac{8(e^*)^2r + 2f^*(n-1)(e^* - 2f^*) - 4e^*f^*r - \sqrt{e^*}r}{8(e^*)^2nr - 4e^*f^*nr - \sqrt{e^*}nr} \end{aligned}$$

translates (33) into the condition

$$\sigma^2 \geq \frac{n \left(-2(-n^2 + n + 2)^2 r^2 - (n-1)^2 n^2 + 2(n-2)(n-1)(n+1)nr \right) + \sqrt[3]{-n - \frac{1}{2}}}{4(-n-1)^{4/3}(n-1)(n(6n+7)+2)r^2} \quad (34)$$

which represents the threshold beyond which individual variance σ^2 can be compressed in the contest-based redistributive agreement. $\square$

Proof of proposition 6. Only the variance stemming from the shocks (28) changes when we move to the non-i.i.d. case relative to the previous proof; condition (30) remains unchanged. The variance of the redistributed resource is now for arbitrary player i

$$\begin{aligned} &\mathbb{V}[(a/n)(y(e_1, \varepsilon_1) + \dots + y(e_n, \varepsilon_n)) + by(e_i, \varepsilon_i)] \\ &= na^2 \mathbb{V}[\varepsilon_i] + (n^2 - n)a^2 \text{Cov}(\varepsilon_i, \varepsilon_j) + ab \mathbb{V}[\varepsilon_i] + (n-1)ab \text{Cov}(\varepsilon_i, \varepsilon_j) \\ &= \alpha^* \frac{\text{Cov}(\varepsilon_i, \varepsilon_j)(n-1) + \mathbb{V}[\varepsilon_i]}{n} \\ &= \alpha^* \frac{\bar{\sigma}^2(n-1) + \sigma^2}{n} \end{aligned} \quad (35)$$

in which we set $a = \alpha^*$ and $b = 1 - \alpha^*$ as determined in (27). Adding the variance from the contest itself (30) as in the previous proof yields the individual equilibrium contest payoff variance

$$\alpha^* \frac{\bar{\sigma}^2(n-1) + \sigma^2}{n} + \mathbb{E}[y(e, \varepsilon)^2] \frac{(\alpha^* - 1)^2(n((2(\beta^* - 1)\beta^* + 1)n - 2) + 2)}{n} \quad (36)$$

which we need to be smaller than standalone income variance σ^2 . This is the case if

$$\sigma^2 \geq \frac{\bar{\sigma}^2(n-1)\alpha^* + \mathbb{E}[y(e, \varepsilon)^2](1 - \alpha^*)^2(2 + n(n(1 + 2(\beta^* - 1)\beta^*) - 2))}{n - \alpha^*}. \quad (37)$$

¹² Since both the variance and output magnitudes depend on the chosen measurement units, it is perhaps preferable to transform (33) into the general, scaling-invariant "signal to noise ratio" requirement that

$$\frac{\mathbb{E}[y^2(e, \varepsilon)]}{\sigma^2} \begin{cases} \leq \frac{(\alpha^* + 1)(n-1)}{(1 - \alpha^*)(n((2(\beta^* - 1)\beta^* + 1)n - 2) + 2)} & \text{if } \sigma^2 \geq 1, \\ > \frac{(\alpha^* + 1)(n-1)}{(1 - \alpha^*)(n((2(\beta^* - 1)\beta^* + 1)n - 2) + 2)} & \text{if } \sigma^2 < 1. \end{cases} \quad (32)$$

Substituting our standard example parameters gives

$$\sigma^2 \geq \frac{\bar{\sigma}^2(n-1)(n+1)}{2n^2-1} + \frac{n^2 \mathbb{E}[y(e, \varepsilon)^2] \left(2(-n^2+n+2)^2 r^2 + (n-1)^2 n^2 - 2(n-2)(n-1)(n+1)nr \right)}{2(n+1)^2(2n+1)(2n^2-1)r^2} \quad (38)$$

which is equal to the condition $\sigma^2 \geq \frac{3}{7}\bar{\sigma}^2 + \frac{8}{315r^2} \mathbb{E}[y(e, \varepsilon)^2]$ for the case of two players which again implies that the agreement compresses variance if the standalone variance exceeds $.0018 + .43\bar{\sigma}^2$. $\square$

Proof of proposition 7. Since each individual output $y(e_i, \varepsilon_i)$ has variance σ^2 and covariance of two distinct variables σ_{ij} , the variance of the prize pool is (for equilibrium α)

$$\begin{aligned} \mathbb{V}[u_i(\mathbf{e}, \mathbf{f}; \varepsilon)] &= \mathbb{V}[\alpha y(e_i, \varepsilon_i)] + \mathbb{V}\left[(1-\alpha)\frac{1}{n}\sum_{i=1}^n y(e_i, \varepsilon_i)\right] + 2\alpha(1-\alpha)\frac{1}{n}\text{Cov}\left(y(e_i, \varepsilon_i), \sum_{i=1}^n y(e_i, \varepsilon_i)\right) \\ &= \alpha^2 \mathbb{V}[y(e_i, \varepsilon_i)] + (1-\alpha)^2 \frac{1}{n^2} \sum_{i=1}^n \mathbb{V}[y(e_i, \varepsilon_i)] + 2\alpha(1-\alpha)\frac{1}{n}\text{Cov}\left(y(e_i, \varepsilon_i), \sum_{i=1}^n y(e_i, \varepsilon_i)\right) \\ &\quad + 2\frac{(1-\alpha)^2}{n^2} \text{Cov}\left(\sum_{i=1}^n y(e_i, \varepsilon_i), \sum_{j=1}^n y(e_j, \varepsilon_j)\right) \\ &= \sigma^2 \left[\alpha^2 + (1-\alpha)^2 \frac{1}{n} \right] + \alpha(1-\alpha)\frac{1}{n} \sum_{i \neq j} \sigma_{ij} + 2\frac{(1-\alpha)^2}{n} \sum_{1 \leq i < j \leq n} \sigma_{ij} \\ &\leq \sigma^2 \left[\alpha^2 + (1-\alpha)^2 \frac{1}{n} \right] + 2\alpha(1-\alpha)\frac{n-1}{n}\sigma^2 + \frac{(1-\alpha)^2}{n}n(n-1)\sigma^2 \\ &= \frac{\sigma^2}{n} [2\alpha^2 - 2\alpha + n] \\ &< \sigma^2 \quad \forall \alpha \in (0, 1). \end{aligned} \quad \square$$

Proof of proposition 8. Let us first consider independent shocks. For equilibrium α_i, β_i ,

$$\begin{aligned} \mathbb{V}[u_i(\mathbf{e}^*, \mathbf{f}^*); \varepsilon] &= \mathbb{V}[\alpha_i y(e_i^*, \varepsilon_i) + p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i[(1-\alpha_i)y(e_i^*, \varepsilon_i) \\ &\quad + (1-\alpha_j)y(e_j^*, \varepsilon_j)] + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)[(1-\alpha_i)y(e_i^*, \varepsilon_i) + (1-\alpha_j)y(e_j^*, \varepsilon_j)]] \\ &= [\alpha_i + (p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j))(1-\alpha_i)]^2 \sigma^2 \\ &\quad + [p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)]^2 (1-\alpha_j)^2 \sigma^2 \\ &= \alpha_i^2 \sigma^2 + 2\alpha_i(1-\alpha_i)[p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)]\sigma^2 \\ &\quad + [p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1-p_i(\mathbf{e}^*, \mathbf{f}^*))(1-\beta_j)]^2 ((1-\alpha_i)^2 + (1-\alpha_j)^2) \sigma^2 \\ &\leq \alpha_i^2 \sigma^2 + 2\alpha_i(1-\alpha_i)\beta_i \sigma^2 + ((1-\alpha_i)^2 + (1-\alpha_j)^2)[p_i(\mathbf{e}^*, \mathbf{f}^*)^2(-1+\beta_i+\beta_j)^2 \\ &\quad + 2p_i(\mathbf{e}^*, \mathbf{f}^*)(1-\beta_j)(-1+\beta_i+\beta_j) + (1-\beta_j)^2] \sigma^2 \\ &\leq \{\alpha_i^2 + 2\alpha_i(1-\alpha_i)\beta_i + ((1-\alpha_i)^2 + (1-\alpha_j)^2)\beta_i^2\} \sigma^2. \end{aligned}$$

Then, $\mathbb{V}[u_i(\mathbf{e}^*, \mathbf{f}^*); \varepsilon] < \sigma^2$ for all i if and only if

$$1 > \beta_i^2(1-\alpha_j)^2 + (\beta_i - \alpha_i\beta_i - \alpha_i)^2 + 4\alpha_i\beta_i(1-\alpha_i). \quad (39)$$

Under non-independent shocks, the covariance term has to be added

$$\begin{aligned} & \text{Cov}([\alpha_i + A(1 - \alpha_i)]y(e_i^*, \varepsilon_i), A(1 - \alpha_j)y(e_j^*, \varepsilon_j)) \\ &= [\alpha_i + A(1 - \alpha_i)]A(1 - \alpha_j) \text{Cov}[y(e_i^*, \varepsilon_i), y(e_j^*, \varepsilon_j)] \end{aligned} \quad (40)$$

$$\begin{aligned} &\leq [\alpha_i + \beta_i(1 - \alpha_i)]\beta_i(1 - \alpha_j)\sigma_{ij} \\ &\leq [\alpha_i + \beta_i(1 - \alpha_i)]\beta_i(1 - \alpha_j)\sigma^2 \end{aligned} \quad (41)$$

with $A = [p_i(\mathbf{e}^*, \mathbf{f}^*)\beta_i + (1 - p_i(\mathbf{e}^*, \mathbf{f}^*))(1 - \beta_j)]$. As for the independent shocks, we use the fact $A \leq \beta_i$ (equations (40) & (41)). Then, using the condition (39) for independent shocks we get

$$1 > \beta_i(3 - 2\alpha_i - \alpha_j)(2\alpha_i + \beta_i - \beta_i\alpha_j) + (\beta_i - \alpha_i\beta_i - \alpha_i)^2 \quad (42)$$

which establishes our claim. $\square$

Proof of proposition 9. Assume that M:(22) holds (otherwise we are done), where $u^{\text{rm}}(\cdot)$ and $u^{\text{fr}}(\cdot)$ are both linear functions in their respective arguments ε^{rm} and ε^{fr} . From theorems 1 and 2, it follows that $\varepsilon^{\text{fr}} \leq_{\text{icv}} \varepsilon^{\text{rm}}$ such as $\mathbb{E}[v(\varepsilon^{\text{fr}})] \leq \mathbb{E}[v(\varepsilon^{\text{rm}})]$ for all increasing and concave functions v . Consequently, there exists a sufficiently concave, increasing function v such that $\mathbb{E}[v(u^{\text{rm}}(\mathbf{e}^*, \mathbf{f}^*; \varepsilon^{\text{rm}}))] \geq \mathbb{E}[v(u^{\text{fr}}(\tilde{\mathbf{e}}, \tilde{\mathbf{f}}; \varepsilon^{\text{fr}}))]$. $\square$

Appendix C: Stochastic Dominance Relations

This appendix provides analytical tools and new results on stochastic dominance which are useful to establish proposition 9.

Definition 1 (Shaked & Shanthikumar (1994)). *Let X and Y be two random variables. X is said to be smaller than Y in the increasing concave order and denoted $X \leq_{\text{icv}} Y$ if and only if*

$$\mathbb{E}[v(X)] \leq \mathbb{E}[v(Y)] \quad (43)$$

for all increasing, concave functions $v(\cdot)$.

In the economics literature, increasing concave orders are typically referred to as *second order stochastic dominance*. Similarly, increasing concave orders with equal means are usually called *mean preserving spreads*. An alternative, equivalent definition is:

Definition 2 (Shaked & Shanthikumar (1994)). *Let X and Y be two random variables with F_X and F_Y their continuous cumulative functions. $X \leq_{\text{icv}} Y$ if and only if*

$$\int_{-\infty}^z F_X(t)dt \geq \int_{-\infty}^z F_Y(t)dt. \quad (44)$$

The following result generalises theorem 5 of Müller (2001) for univariate elliptical distributions and increasing concave orders. The proof is provided for completeness and is an adaptation of that in Müller (2001).

Theorem 1. Let $X \sim \mathcal{L}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{L}(\mu_y, \sigma_y^2)$, which are two elliptically distributed random variables. If $\sigma_y^2 \leq \sigma_x^2$ and $\mu_y \leq \mu_x$ then $X \leq_{icv} Y$.

Proof. Given the properties of elliptical distributions, the distribution of X is equal to that of $Y + Z$ with $Z \sim \mathcal{L}(\mu_x - \mu_y, \sigma_x^2 - \sigma_y^2)$.¹³ Then,

$$\begin{aligned} \mathbb{E}[v(X)] &= \mathbb{E}[v(Y + Z)] \\ &= \mathbb{E}[\mathbb{E}[v(Y + Z)|Y]] \end{aligned} \tag{45}$$

$$\leq \mathbb{E}[v(Y + \mathbb{E}[Z])] \tag{46}$$

$$\leq \mathbb{E}[v(Y)]. \tag{47}$$

The Jensen inequality for concave functions leads to (46). Then, using $\mathbb{E}[Z] \leq 0$ (47) follows. $\square$

Theorem 2. Let $X \sim \mathcal{L}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{L}(\mu_y, \sigma_y^2)$, which are two elliptically distributed random variables such that $\mu_y = \mu_x = 0$. Then the following statements are equivalent: (i) $X \leq_{icv} Y$, and (ii) $\sigma_y^2 \leq \sigma_x^2$.

Proof. Every probability density $(F_i)'$ is symmetric and continuous. Zero mean implies that the symmetry of the probability density is with respect to the ordinate. Then, every cumulative distribution has an inflection point at zero. Furthermore, X has a probability density with bigger tails than Y because of the ranking of the variance. Consequently of the tails and the inflection point, the cumulative distribution of X is above (below) the one of Y for all value inferior (superior) to zero such that the surfaces between the two cumulative distributions before and after zero are equal. Then,

$$\int_{-\infty}^z F_X(t)dt \geq \int_{-\infty}^z F_Y(t)dt \tag{48}$$

which implies that $X \leq_{icv} Y$. Notice that this implication is true by application of Theorem 1.

Moreover, it is well-known that if $X \leq_{icv} Y$ and $\mathbb{E}[X] = \mathbb{E}[Y]$ then $\sigma_y^2 \leq \sigma_x^2$ follows (see, for instance, Rothschild & Stiglitz (1970)). $\square$

Appendix D: Further examples

Section 5 of the main paper illustrates the intuition of our principal result: the characterisation of efficient equilibria. This section provides a sequence of further examples for the other results of the paper in a similar environment.

To get a feeling for the magnitudes implied by our example mechanism we plug in the 2011 global GDP of \$80tr, or, among two identical players, \$40tr GDP per player. Thus, our proposed mechanism collects $P = (2/5)80 = \$24.4\text{tr}$ or \$16tr from each player. The following table lists the redistribution implied by the efficient mechanism. Depending on the precision of monitoring r , the mechanism parameters and transfers are

¹³ See Fang et al. (1987) or Landsman & Valdez (2003) for details.

r	β	$1 - \beta$	1 st	2 nd	transfer	%
1	$\frac{2}{3}$	$\frac{1}{3}$	\$21.3tr	\$10.7tr	$\pm \$5.3tr$	$\pm 13.3\%$
2	$\frac{7}{12}$	$\frac{5}{12}$	\$18.7tr	\$13.2tr	$\pm \$2.7tr$	$\pm 6.7\%$
.	...	...	...	...	...	...
5	$\frac{8}{15}$	$\frac{7}{15}$	\$17.0tr	\$15.0tr	$\pm \$1.0tr$	$\pm 2.7\%$
.	...	...	...	...	...	...
11	$\frac{17}{33}$	$\frac{16}{33}$	\$16.5tr	\$15.5tr	$\pm \$0.5tr$	$\pm 1.2\%$

where the rightmost column's percentages are taken from the total symmetric \$40tr GDP. Our sufficient existence condition derived in proposition 2 guarantees existence up to and including $r = 5$. Actual existence, however, is only lost for $r > 11$. At this monitoring level, a transfer of 1.2% of GDP is sufficient to implement both efficient abatement and efficient production.¹⁴

As pointed out in the model section and the discussion of the sharing mechanism in section 4.2.2, we can alternatively employ the interpretation of where each player gets exactly a contractible 'share' $p_i(\mathbf{f})$ of agreement output. Under this mathematically equivalent interpretation, while the incentives incorporated in the mechanism ensure efficient efforts along both dimensions, all symmetric equilibrium transfers cancel out.

D.1 Example of the asymmetric contest

As an illustration of the asymmetric modelling idea of section 3.3 in the main text, the following example provides an analytical characterisation of the efficient equilibrium. Consider the following three-players concretisation of M:(10)¹⁵

$$\begin{aligned}
& \alpha_i \underbrace{(e_i)}_{=y_i(e_i)} + \left(\underbrace{\frac{f_i^r}{f_1^r + f_2^r + f_3^r}}_{=p_i^1(\mathbf{f})} \beta_i + \underbrace{\frac{f_2^r}{f_1^r + f_2^r + f_3^r}}_{=p_2^1(\mathbf{f})} \frac{1 - \beta_2}{2} + \underbrace{\frac{f_3^r}{f_1^r + f_2^r + f_3^r}}_{=p_3^1(\mathbf{f})} \frac{1 - \beta_3}{2} \right) P \\
& - s_i \underbrace{(G)^2}_{=m(G)} - \underbrace{\gamma_i \frac{e_i^2}{2}}_{=c_{i,e}(e_i)} - \underbrace{\delta_i \frac{f_i^2}{2}}_{=c_{i,f}(f_i)}
\end{aligned} \tag{49}$$

in which $P = (1 - \alpha_1)(e_1) + (1 - \alpha_2)(e_2) + (1 - \alpha_3)(e_3)$ and $\gamma_i, \delta_i > 0$.¹⁶

We now derive the asymmetric tax rates α_i^* and redistributive shares β_i^* , $i = 1, 2, 3$, which induce fully efficient productive and abatement equilibrium efforts in the asymmetric contest (49) while balancing the agreement's budget on and off the equilibrium path. The first-order, necessary

¹⁴ The European Union plans to dedicate up to 1/5 of its budget to the fight against global warming. This will represent up to 180 billions euros of climate related project spending over a seven-year period. For details see http://ec.europa.eu/clima/policies/budget/index_en.htm.

¹⁵ The two players asymmetric case in the example setup is rather special since $\beta_1 + (1 - \beta_2) = 1$ implies $\beta_1 = \beta_2$. Hence, in order to lend itself to a solution, the two players case requires success function slopes of precisely

$$\frac{\partial}{\partial f_i} p_i(\mathbf{f}) = \frac{(1 - s_i)c'_f(f_i)}{(\beta_1 + \beta_2 - 1)P}.$$

Cases with higher numbers of players than two do not imply equal prize shares and do not induce complications.

¹⁶ We use cost heterogeneity in our development of the asymmetric case. Since only the equalisation of marginal benefits and costs matter for the argument, the asymmetries can be equally thought of on the production side.

condition for (49) with respect to e_i is

$$\alpha_i = \frac{(\alpha_i - 1) \left(2\beta_i f_i^r + \left(\sum_{j \neq i} f_j^r (1 - \beta_j) \right) \right)}{2(f_1^r + f_2^r + f_3^r)} + 2s_i G + \gamma_i e_i, \quad (50)$$

the first-order condition for (49) with respect to f_i is

$$2s_i G = \frac{r f_i^{r-1} \left(\sum_{i=1}^3 e_i (\alpha_i - 1) \right) \left(\sum_{i=1}^3 f_j^r (2\beta_i - \beta_j - 1) \right)}{2(f_1^r + f_2^r + f_3^r)^2} + \delta_i f_i \quad (51)$$

for $i = 1, 2, 3$. Setting efforts equal to the efficient quantities M:(11) from the planner's problem

$$\begin{aligned} e_i^* &= \frac{\left(\prod_{j \neq i} \gamma_j \right) \left(2 \left(\prod_{j \neq i} \delta_j \right) + \delta_i \left(2 \left(\sum_{j \neq i} \delta_j \right) + \prod_{j \neq i} \delta_j \right) \right)}{\gamma_1 (\gamma_2 (\gamma_3 (\delta_1 (\delta_2 (\delta_3 + 2) + 2\delta_3) + 2\delta_2 \delta_3) + 2\delta_1 \delta_2 \delta_3) + 2\gamma_3 \delta_1 \delta_2 \delta_3) + 2\gamma_2 \gamma_3 \delta_1 \delta_2 \delta_3}, \\ f_i^* &= \frac{2 \left(\left(\prod_{j \neq i} \gamma_j \right) + \gamma_i \left(\sum_{j \neq i} \gamma_j \right) \right) \left(\prod_{j \neq i} \delta_j \right)}{\gamma_1 (\gamma_2 (\gamma_3 (\delta_1 (\delta_2 (\delta_3 + 2) + 2\delta_3) + 2\delta_2 \delta_3) + 2\delta_1 \delta_2 \delta_3) + 2\gamma_3 \delta_1 \delta_2 \delta_3) + 2\gamma_2 \gamma_3 \delta_1 \delta_2 \delta_3}. \end{aligned} \quad (52)$$

We solve these for the parametrisation of $\delta_2 = \delta_3 = 1$, equal shares $s_i = 1/3$, and the case of $r = 1$ in order to avoid equilibrium existence problems. Further generalisations of this contest can typically only be solved numerically. This yields the mechanism

$$\begin{aligned} \alpha_1^* &= \frac{1}{3\gamma_2 \gamma_3 (4\delta_1 - 1) (5\delta_1 + 2) (\gamma_1 (\gamma_2 (5\gamma_3 \delta_1 + 2\gamma_3 + 2\delta_1) + 2\gamma_3 \delta_1) + 2\gamma_2 \gamma_3 \delta_1)} \\ &\left(2\gamma_1^2 (4\delta_1 - 1) (9\delta_1^2 - 2) (\gamma_2 + \gamma_3)^2 \right. \\ &\quad \left. - 2\gamma_2 \gamma_3 \left(\sqrt{(1 - 4\delta_1)^2 (\gamma_1 (\gamma_2 + \gamma_3) + \gamma_2 \gamma_3) \left(\gamma_1 (2 - 9\delta_1^2)^2 (\gamma_2 + \gamma_3) + \gamma_2 \gamma_3 (\delta_1 (\delta_1 + 4) + 2)^2 \right)} \right. \right. \\ &\quad \left. \left. - \gamma_2 \gamma_3 (4\delta_1 - 1) (9\delta_1^2 - 2) \right) \right. \\ &\quad \left. + \gamma_1 \left(-2\gamma_2 \sqrt{(1 - 4\delta_1)^2 (\gamma_1 (\gamma_2 + \gamma_3) + \gamma_2 \gamma_3) \left(\gamma_1 (2 - 9\delta_1^2)^2 (\gamma_2 + \gamma_3) + \gamma_2 \gamma_3 (\delta_1 (\delta_1 + 4) + 2)^2 \right)} \right. \right. \\ &\quad \left. \left. - 2\gamma_3 \sqrt{(1 - 4\delta_1)^2 (\gamma_1 (\gamma_2 + \gamma_3) + \gamma_2 \gamma_3) \left(\gamma_1 (2 - 9\delta_1^2)^2 (\gamma_2 + \gamma_3) + \gamma_2 \gamma_3 (\delta_1 (\delta_1 + 4) + 2)^2 \right)} \right. \right. \\ &\quad \left. \left. + \gamma_2 \gamma_3 (4\delta_1 - 1) (\gamma_2 (3\gamma_3 (5\delta_1 + 2)^2 + 36\delta_1^2 - 8) + 4\gamma_3 (9\delta_1^2 - 2)) \right) \right) \right), \end{aligned} \quad (53)$$

$$\begin{aligned}
\alpha_2^* = \alpha_3^* = & \frac{1}{3\gamma_1(4\delta_1 - 1)(5\delta_1 + 2)(\gamma_2 + \gamma_3)(\gamma_1(\gamma_2(5\gamma_3\delta_1 + 2\gamma_3 + 2\delta_1) + 2\gamma_3\delta_1) + 2\gamma_2\gamma_3\delta_1)} \\
& \left(\gamma_1^2(4\delta_1 - 1)(\gamma_2 + \gamma_3) \left(\gamma_2(3\gamma_3(5\delta_1 + 2)^2 + \delta_1(\delta_1 + 4) + 2) + \gamma_3(\delta_1(\delta_1 + 4) + 2) \right) \right. \\
& \quad \left. + \gamma_1(\gamma_2 + \gamma_3) \left(2\gamma_2\gamma_3(4\delta_1 - 1)(\delta_1(\delta_1 + 4) + 2) \right. \right. \\
& \quad \left. \left. - \sqrt{(1 - 4\delta_1)^2(\gamma_1(\gamma_2 + \gamma_3) + \gamma_2\gamma_3) \left(\gamma_1(2 - 9\delta_1^2)^2(\gamma_2 + \gamma_3) + \gamma_2\gamma_3(\delta_1(\delta_1 + 4) + 2)^2 \right)} \right) \right. \\
& \quad \left. + \gamma_2\gamma_3 \left(\gamma_2\gamma_3(4\delta_1 - 1)(\delta_1(\delta_1 + 4) + 2) \right. \right. \\
& \quad \left. \left. - \sqrt{(1 - 4\delta_1)^2(\gamma_1(\gamma_2 + \gamma_3) + \gamma_2\gamma_3) \left(\gamma_1(2 - 9\delta_1^2)^2(\gamma_2 + \gamma_3) + \gamma_2\gamma_3(\delta_1(\delta_1 + 4) + 2)^2 \right)} \right) \right) \right), \tag{54}
\end{aligned}$$

$$\begin{aligned}
\beta_1^* = & \frac{1}{(\delta_1 - 1)(4\delta_1 - 1)(\gamma_2\gamma_3(\delta_1(7\delta_1 + 10) + 4) - 2\gamma_1(\delta_1(19\delta_1 + 4) - 2)(\gamma_2 + \gamma_3))} \\
& \left(\delta_1 \right. \\
& \quad \left. + 2 \right) \left(-\sqrt{(1 - 4\delta_1)^2(\gamma_1(\gamma_2 + \gamma_3) + \gamma_2\gamma_3) \left(\gamma_1(2 - 9\delta_1^2)^2(\gamma_2 + \gamma_3) + \gamma_2\gamma_3(\delta_1(\delta_1 + 4) + 2)^2 \right)} \right. \\
& \quad \left. - \gamma_1\delta_1(4\delta_1 - 1)(\delta_1(9\delta_1 - 20) - 10)(\gamma_2 + \gamma_3) + \gamma_2\gamma_3\delta_1(2\delta_1 + 1)(3\delta_1 + 4)(4\delta_1 - 1) \right), \tag{55}
\end{aligned}$$

$$\begin{aligned}
\beta_2^* = \beta_3^* = & \frac{1}{(\delta_1 - 1)(\gamma_2\gamma_3(\delta_1(7\delta_1 + 10) + 4) - 2\gamma_1(\delta_1(19\delta_1 + 4) - 2)(\gamma_2 + \gamma_3))} \\
& \left(-\sqrt{(1 - 4\delta_1)^2(\gamma_1(\gamma_2 + \gamma_3) + \gamma_2\gamma_3) \left(\gamma_1(2 - 9\delta_1^2)^2(\gamma_2 + \gamma_3) + \gamma_2\gamma_3(\delta_1(\delta_1 + 4) + 2)^2 \right)} \right. \\
& \quad \left. + \gamma_1(2\delta_1 + 1)(\delta_1(\delta_1 + 8) - 2)(\gamma_2 + \gamma_3) + \gamma_2\gamma_3(\delta_1(\delta_1(17\delta_1 + 8) - 2) - 2) \right). \tag{56}
\end{aligned}$$

For the particular values $\gamma_1 = 3/4$, $\gamma_2 = 1/2$, $\gamma_3 = 1/3$, $\delta_1 = 5/4$ this yields, for instance,

$$\alpha_1^* = 0.408, \alpha_2^* = \alpha_3^* = 0.304, \beta_1^* = 0.812, \beta_2^* = \beta_3^* = 0.922.$$

The usual graphs confirm the existence of this asymmetric equilibrium.

D.2 Example of asymmetric *relative* reductive efforts per-unit-GDP

This example illustrates the use of an asymmetric contest based on GDP-normalised efforts as in M:(14). In our standard square-root/quadratic example environment, efficient efforts (e^*, f^*) are

independent of the contest technology and thus given by the asymmetric but otherwise unchanged three-players version of M:(23) as¹⁷

$$\begin{aligned} (e^*, f^*) \in \arg \max_{(e, f)} u(e, f) = \\ y(e_1) + y(e_2) + y(e_3) - \\ (e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \\ \gamma_1 (e_1^2 + f_1^2) - \gamma_2 (e_2^2 + f_2^2) - \gamma_3 (e_3^2 + f_3^2) \end{aligned} \Leftrightarrow \begin{cases} e_1^* = 0.513, f_1^* = 0.535, \\ e_2^* = 0.359, f_2^* = 0.267, \\ e_3^* = 0.288, f_3^* = 0.178. \end{cases} \quad (57)$$

Again for simplicity, we choose linear cost asymmetries $\gamma_3 = 1$, $\gamma_2 = 2/3$, $\gamma_1 = 1/3$ to differentiate players but asymmetric productive capability could be modelled in exactly the same way. Finally, we use relative pollution shares of $s_1 = 4/12$, $s_2 = 3/12$, $s_3 = 5/12$. For simplicity, we only present numerical results in this section.

For the three-players asymmetric contest, the redistribution pool is $P = \sum_{i=1}^3 (1 - \alpha_i)(\sqrt{e_i})$. This setup results in the three players' objectives

$$\begin{aligned} u_1(e, f) &= \alpha_1 y(e_1) + p_1^1(e, f) \beta_1 P + (1 - p_1^1(e, f)) \left(\frac{x_2^r}{x_2^r + x_3^r} \frac{1 - \beta_2}{2} + \frac{x_3^r}{x_2^r + x_3^r} \frac{1 - \beta_3}{2} \right) P - \\ &\quad s_1 (e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_1 (e_1^2 + f_1^2), \\ u_2(e, f) &= \alpha_2 y(e_2) + p_2^1(e, f) \beta_2 P + (1 - p_2^1(e, f)) \left(\frac{x_1^r}{x_1^r + x_3^r} \frac{1 - \beta_1}{2} + \frac{x_3^r}{x_1^r + x_3^r} \frac{1 - \beta_3}{2} \right) P - \\ &\quad s_2 (e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_2 (e_2^2 + f_2^2), \\ u_3(e, f) &= \alpha_3 y(e_3) + p_3^1(e, f) \beta_3 P + (1 - p_3^1(e, f)) \left(\frac{x_1^r}{x_1^r + x_2^r} \frac{1 - \beta_1}{2} + \frac{x_2^r}{x_1^r + x_2^r} \frac{1 - \beta_2}{2} \right) P - \\ &\quad s_3 (e_1 + e_2 + e_3 - f_1 - f_2 - f_3)^2 - \gamma_3 (e_3^2 + f_3^2). \end{aligned} \quad (58)$$

The corresponding six first-order conditions induce the set of efficient efforts (57) as individual global maxima for the following set of design parameters $\langle \alpha, \beta, r^* = 2 \rangle$ ¹⁸

$$\begin{aligned} \alpha_1^* &= 0.610, & \alpha_2^* &= 0.740, & \alpha_3^* &= 0.810, \\ \beta_1^* &= 0.767, & \beta_2^* &= 0.518, & \beta_3^* &= 0.460. \end{aligned} \quad (60)$$

The below figure confirms that these identify an equilibrium for the game where a ratio of asymmetric strategic variables is chosen.

D.3 Agreement formation

Our argument in proposition 3 uses the maximal threat $C' = \langle \alpha' = 1, \beta'; p(f)' \rangle$ to show that free-riding can always be discouraged. This extreme case, however, renders the agreement wholly ineffective if a punishment becomes necessary. The purpose of the following example is to show that

¹⁷ Since the two-players asymmetric example with $\beta_1 + (1 - \beta_2) = 1 \Leftrightarrow \beta_1 = \beta_2$ is still 'too symmetric' to illustrate the workings of the asymmetric mechanism we switch into a three players example in this section.

¹⁸ In the setup of section 4.2, the efficiency inducing parameter set of the symmetric version of this 'normalised' contest based on $x_i = f_i/y(e_i)$ is given by

$$\alpha^* = \frac{11}{15}, \quad \beta^* = \frac{1}{2} + \frac{1}{4r}. \quad (59)$$

This is more favourable to the participants than the corresponding parameters under the standard contest M:(27).

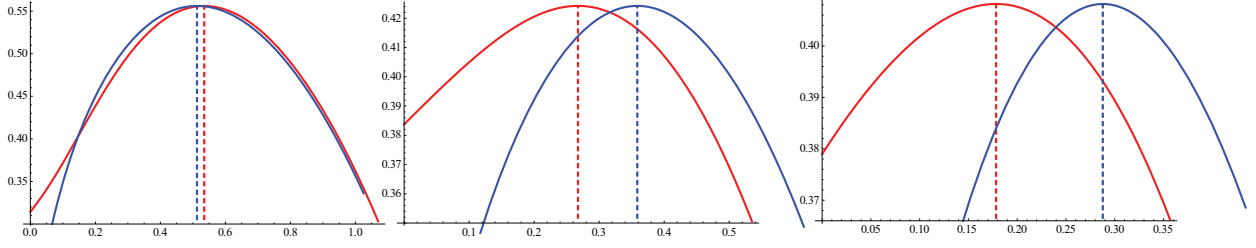


Figure 2: Efficient effort pairs as global maximisers of (58) for player 1's (left), player 2's (centre) and player 3's unilateral deviation utility (right). Red are abatement efforts, blue productive efforts. (The ranges shown contain all maxima.)

such severe measures are not generally needed. Typically, the punishment of a deserter leaves enough freedom to increase abatement levels over those realising under no agreement. As outlined in the previous sections, contract C implements efficient efforts. Consider now a deviation by some player which triggers $C' = \langle \alpha', \beta'; p(\mathbf{f}') \rangle$. Denote the equilibrium agreement utility attained by adhering to C' by $u'(\cdot)$ and the corresponding equilibrium efforts by $e'(\alpha', \beta', p')$, $f'(\alpha', \beta', p')$. By inflicting sufficient damage through $m(\cdot)$, we need to ascertain that free-riding utility $u_i^s(e_i^s, f_i^s)$ —with the agreement members adhering to C' —is smaller than what participation in C gives, i.e.,

$$u_i^s(e_i^s, f_i^s | C') = y(e_i^s) - s_i m(e_i^s + (n-1)e'(\cdot) - f_i^s - (n-1)f'(\cdot)) - c_e(e_i^s) - c_f(f_i^s) \leq u_i(e^*, f^* | C).$$

Since M:(23) implies that efforts e', f' are monotonic in α', β' , payoff $u_i^s(\cdot | C')$ is continuous in α' and β' . Hence there exists an $\alpha' \in (\alpha, 1]$ which ensures the above inequality for suitable β' and $p'(\mathbf{f})$. Consider the case of $n+1$ players in our main example from section 5. Then, full participation efficient efforts are given by

$$e^* = \frac{n+2}{2 \times 2^{1/3} ((2+n)(3+2n)^2)^{1/3}}, \quad f^* = \frac{n+1}{2 \times 2^{1/3} ((2+n)(3+2n)^2)^{1/3}} \quad (61)$$

which are implemented by

$$\alpha^* = \frac{4\sqrt{e^*}(2e^* - f^*)(n+1) - 1}{n}, \quad \beta^* = \frac{1}{n+1} + \frac{2(e^* - 2f^*)f^*}{\sqrt{e^*}r(\alpha - 1)} \quad (62)$$

for the Tullock success function parameterised by r . This determines $u_i(e^*, f^* | C)$. For the deviation utility $u_i^s(e_i^s, f_i^s | C')$, the deviation efforts e^d, f^d are determined by the first-order conditions

$$2e^s = \frac{1}{2\sqrt{e^s}} + \frac{2(f^s + n(f'(\cdot) - e'(\cdot)) - e^s)}{n+1}, \quad f^s = \frac{e^s + n(e'(\cdot) - f'(\cdot))}{n+2} \quad (63)$$

in which $e'(\cdot), f'(\cdot)$ are the agreement equilibrium efforts in the agreement under C' . These functions $e'(\cdot), f'(\cdot)$, and therefore the damage they inflict on the deviator through $m(e^s + ne' - f^s - nf')$, are determined by

$$\alpha' = \frac{n(4\sqrt{e'}(e' + e^s - f^s + 2e'n - f'n) - 1)}{n^2 - 1},$$

$$\beta' = \frac{(e^{2'}(4+8n) - 2f'(n-1)(f' + f^s + 2f'n - e^s) + 2e'(2e^s - 2f^s + f'(n-3)n) - \sqrt{e'}(n+1))}{(4e^{2'}n(1+2n) - 4e'n(-e^s + f^s + f'n) - \sqrt{e'}n(1+n))}.$$

In our example setup, it turns out that participation is individually rational for any number of players. The details for the simplest case of three players (two in the agreement, one outside) are

$$e' = 0.302, f' = 0.195, e^s = 0.314, f^s = 0.132, (e^* = 0.273, f^* = 0.205)$$

for $C' = \langle \alpha' \approx 0.910, \beta' = 1; r' = 1 \rangle$. If there is no agreement, efforts are $e^d = 0.303, f^d = 0.151$, so the 29% increased abatement efforts achieved by the punishment contract are substantial.

D.4 The efficient stochastic mechanism

This section illustrates the idea of variance compression in our usual two players example from section 5. Recall that under the efficient mechanism, player i expects the equilibrium utility from participating in the agreement¹⁹

$$u_i(e^*, f^*; \varepsilon) = \alpha^* y_i(e^*, \varepsilon_i) + \frac{1}{2}P - \frac{1}{2}(2e^* - 2f^*)^2 - (e^*)^2 - (f^*)^2, \quad (64)$$

with $P = (1 - \alpha^*)(y(e^*, \varepsilon_1) + y(e^*, \varepsilon_2))$. The main insurance argument rests on the simple observation that the variance of individual output σ^2 is higher than that of the pooled resource $\tilde{\sigma}^2$ to which all players contribute a share. The intuition is that the risk inherent in individual shocks with zero expectation evens out among several players. This is easiest to see if we assume i.i.d. shocks ε_i for which $\mathbb{V}[\sum_i \varepsilon_i] = \sum_i \mathbb{V}[\varepsilon_i]$, $i = 1, 2$. (In this environment it is safe to drop subscripts on ε .) The variance of the pooled output shocks is then given by (28) in the proof of proposition 5 as

$$\mathbb{V}_\varepsilon(\mathbb{E}(L|\varepsilon)) = \sigma^2 \frac{1 + (\alpha^*)^2(n-1)}{n} = \sigma^2 \frac{1 + (\alpha^*)^2}{2} < \sigma^2. \quad (65)$$

To this variance from the shocks alone we need to add the variance introduced through the contest prize structure. In expectation the contest does not play a role and, therefore, the expected contest payoff is just $\mathbb{E}[L|\varepsilon] = y(e^*, \varepsilon)$. Therefore, the contest variance is given by (30) as

$$\begin{aligned} \mathbb{V}(L|\varepsilon) &= \frac{1}{2}((1 - \alpha)\beta^* 2y(e, \varepsilon) + \alpha^* y(e, \varepsilon) - \mathbb{E}[L|\varepsilon])^2 \\ &\quad + \frac{1}{2}((1 - \alpha^*)(1 - \beta^*)ny(e, \varepsilon) + \alpha^* y(e, \varepsilon) - \mathbb{E}[L|\varepsilon])^2. \end{aligned} \quad (66)$$

From the law of total variance $\mathbb{V}[L] = \mathbb{E}[\mathbb{V}[L|\varepsilon]] + \mathbb{V}[\mathbb{E}[L|\varepsilon]]$, the condition (33) for an agreement member's payoff variance to be smaller than the standalone variance is in the two-players case

$$\sigma^2 \geq \mathbb{E}[y(e, \varepsilon)^2] \frac{2(1 - \alpha^*)(1 - 2\beta^*)^2}{\alpha^* + 1}. \quad (67)$$

Substituting the equilibrium parameters of our standard square-root production and quadratic costs example (27) for $r = 2$ into the above gives the condition $\sigma^2 \geq 0.0039$. When this inequality is satisfied, the agreement equilibrium payoff has a smaller variance than standalone income.

¹⁹ Notice that, since the efficient mechanism uses the ranking only for incentive purposes (based on expectations), the share $(1 - \alpha)y(e_i, \varepsilon_i)$ that players commit to is stochastic.

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