Games and Economic Behavior 151 (2025) 95–107

Contents lists available at ScienceDirect

Games and Economic Behavior

journal homepage: www.elsevier.com/locate/geb

Auctions with signaling bidders: Optimal design and information disclosure ☆

Olivier Bos *, Martin Pollrich

Université Paris-Saclay, ENS Paris-Saclay, Centre for Economics at Paris-Saclay, France

A R T I C L E I N F O

JEL classification:
D44
D82

Keywords:
Optimal auctions
Revenue equivalence
Bayesian persuasion
Information design

A B S T R A C T

We study optimal auctions in a symmetric private values setting, where bidders have signaling concerns: they care about winning the object and a receiver's inference about their type. Signaling concerns arise in various economic situations such as takeover bidding, charity auctions, procurement and art auctions. We show that auction revenue can be decomposed into the standard revenue from the respective auction without signaling concern, and a signaling component. The latter is the bidders' ex-ante expected signaling value net of an endogenous outside option: the signaling value for the lowest type. The revenue decomposition restores revenue equivalence between different auction designs, provided that the same information about bids is revealed. Revealing information about submitted bids affects revenue via the endogenous outside option. In general, revenue is not monotone in information revelation: revealing more information about submitted bids may reduce revenue. We show that any bid disclosure rule allowing to distinguish whether a bidder submitted a bid or abstained from participation minimizes the outside option, and therefore maximizes revenue.

1. Introduction

Since 1945, the *Hospices de Beaune*,[1] in Burgundy (France), organizes an annual wine auction to raise money for local retirement houses and hospitals. In a special segment—the "pièce des Présidents"—a few barrels of wine are auctioned with the help of celebrities. Naturally, this segment draws considerable attention by the media, with extended press coverage. In the 2017 "pièce des Présidents" auction two barrels of *Corton Clos du Roi Grand Cru* were sold at a total price of €410,000. During the regular auction, the same wine realized prices ranging from €30,000 to €40,000 per barrel. Roughly speaking, public attention increased the price per barrel by

☆ A previous version circulated as "Optimal Auctions with Signaling Bidders". We are grateful for helpful comments and suggestions by an anonymous advisory editor and two anonymous referees. We would like to thank Federico Vaccari for its thoughtful discussion at the Oligo Workshop. We also thank participants in various seminars and workshops including University of Cyprus, University of Duisburg, 1st KIT-Paris2-ZEW Workshop on Market Design, INFORMS Annual Meeting, the 4th Internal Conference of the CRC TR 224, 2022 European Workshop on Economic Theory, Oligo Workshop 2023, OM Workshop: Auctions and Mechanism Design. Financial support from the ANR SIGNAL (ANR-19-CE26-0009) is gratefully acknowledged. Martin Pollrich gratefully acknowledges financial support from the Deutsche Forschungsgemeinschaft through CRC TR 224 (Project B01).

* Corresponding author.
E-mail addresses: olivier.bos@ens-paris-saclay.fr (O. Bos), martin.pollrich@gmail.com (M. Pollrich).

[1] https://www.beaune-tourism.com/discover/hospices-de-beaune-wine-auction.

https://doi.org/10.1016/j.geb.2025.02.012
Received 7 October 2022

Available online 28 February 2025

500%.[2] This wine auction is but one example of an auction where bidders have signaling concerns, i.e., bidders care about the object at sale but also about how they are perceived by others. For example takeover bidding is affected by the bidding firms' managers career concerns. The managers' future compensation is (partly) influenced by the inference potential employers draw from their performance in the takeover auction (Giovannoni and Makris, 2014). Also, competing firms pay the merger via issuing equity, and the value of equity depends on the market's belief about the value of the merged entity. The latter is the bidders' private information and imperfectly revealed during the auction (Liu, 2012). Signaling concerns are also present in procurement auctions with bidder qualification (Wan and Beil, 2009; Wan et al., 2012). The bidders' performance in past tenders serves as a signal of their intrinsic quality, affecting their chance for being qualified for future tenders. Bidding in license or spectrum auctions is carefully observed by the stock market. The auction outcome is highly relevant for later competition in the market and in itself for the financial well-being of the bidding companies.[3] Finally, Mandel (2009) identifies signaling as an important aspect for buying and investing in artwork.

In this manuscript, we study auction and information design when bidders care about the inference outsiders draw about their type. Each bidder values winning the object. In addition, each bidder has a signaling concern where she cares about the posterior belief about her type (i.e., her valuation for the object at sale). In our setting the seller runs an auction and publicly announces the winner. The auctioneer has two design tools to capitalize on the bidders' willingness to pay for the object and their signaling concern. First, as in standard auction design, the payment rule specifying each bidder's payment as a function of the submitted bids, e.g., first or second price auction. The second design tool is a disclosure rule, revealing information about the submitted bids.[4] Such disclosure can range from no disclosure, where no further information is revealed, to fully disclosing all submitted bids alongside the bidders' identities.

We show a version of the revenue equivalence principle, stating that for a fixed disclosure rule the auctioneer's revenue is the same across all payment rules, i.e., auction designs. This result holds due to additive separability of the bidders' utility, which is the sum of her valuation for the object and the utility derived from signaling. However, the auctioneer's revenue does depend on the employed disclosure rule. Bidders have an endogenous outside option, arising from the (expected) signaling value a bidder realizes by not submitting a bid. Disclosure rules differ in the value of this endogenous outside option, and therefore in revenue. In particular, disclosing *more information* about bids can *decrease* revenue. To maximize revenue, the auctioneer strives at reducing the value of the outside option to its bare minimum. This corresponds to a disclosure rule that allows outsiders to distinguish between bidders that submitted a bid but did not win the object, from bidders that did not even submit a bid. As an example for such a disclosure rule, the auctioneer may run a full transparent auction in which all bidding is public. Also an English auction, in which the identities of active bidders are public reveals the optimal amount of information.

Information design is implicit in many real-world auctions, where laws govern the disclosure of information about submitted bids. Regulations for takeover bidding require public bids, that is every submitted bid gets publicly disclosed.[5] In contrast, in private procurement all details of the bidding process are treated as a trade secret. Merely the identity of the winning bidder becomes public, no information on bids and payments is revealed to the public. These examples capture the two extremes of information disclosure: full disclosure (takeover bidding) and no disclosure (private procurement). But information disclosure can also take forms in between these extremes. In public procurement regulations call for concealing individual bids, but the final price has to be published.[6] Because the final price is itself a function of submitted bids, there is some implicit information disclosure which depends on the selected payment rule. In a first-price auction with public revelation of the winner's payment, outsiders infer the winner's bid from her payment. At the same time there is only noisy inference on losers' bids. Similarly, in a second-price auction with revelation of the winner's payment, outsiders observe the highest losing bid. This renders inference on *all* bidders noisy, because the identity of the highest losing bidder remains unknown. An all-pay auction provides an example of an auction format where information revelation is implicit (via prices) but still corresponds to full information disclosure as in the case of takeover bidding mentioned earlier.

In their seminal contributions Myerson (1981) and Riley and Samuelson (1981) show that—absent signaling concerns—every standard auction yields the same revenue. This is not necessarily the case when bidders care for signaling. Giovannoni and Makris (2014); Bos and Truyts (2021) study environments where different auction formats and bid disclosure policies yield different auction revenue, while for instance Goeree (2003); Molnar and Virag (2008); Katzman and Rhodes-Kropf (2008) find revenue equivalence in their respective settings. In this manuscript we study the simultaneous design of the auction (i.e., the payment rule) and information disclosure. We ask which information disclosure is optimal, and how the two design tools interact, i.e., for which disclosure rules does revenue equivalence obtain.

---

[2] Similar patterns arose in the previous years. Data for 2016 and 2017 are available at http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/3869/14085/version/1/file/catalogue_resultats_2016.pdf and http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/4248/15476/version/1/file/Vente+des+vins+-+Catalogue+des+r%C3%A9sultats+2017.pdf.

[3] The 5G spectrum auction held in Germany in 2019 was one of the most competitive auctions in European countries. It raised €6.5 billions. Drillisch Netz AG became the new fourth mobile operator with two bids of €1.07 billion together for 70 MHz of spectrum. In the meantime its share price increased by 11%. See https://www.reuters.com/article/us-germany-telecoms/germany-raises-6-55-billion-euros-in-epic-5g-spectrum-auction-idINKCN1TD27D?edition-redirect=in and https://www.bundesnetzagentur.de/SharedDocs/Pressemitteilungen/EN/2019/20190612_spectrumauctionends.html.

[4] Formally, the auctioneer designs a *disclosure rule*, which maps a vector bids into publicly signals. See the model section for a formal definition and further examples.

[5] See Article 6 of Directive 2004/25/EC of the European Parliament and of the Council of 21 April 2004 on takeover bids: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32004L0025&from=EN (last accessed September 3rd 2024).

[6] See Articles 21 and 22 as well as Annex V Part D of Directive 2014/24/EU of the European Parliament and of the Council of 26 February 2014 on public procurement: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:02014L0024-20200101&from=EN (last accessed September 3rd 2024).

Auctions with signaling concerns have been recently investigated by Giovannoni and Makris (2014) and Bos and Truyts (2021).[7] While these papers focus on specific auction formats and disclosure rules, our analysis covers all standard auctions and arbitrary bid-disclosure policies. Giovannoni and Makris consider first- and second price auctions that reveal the winner's identity together with four disclosure policies: no further information, disclosure of the highest bid, disclosure of the second highest bid, and disclosure of all bids. As confirmed by our analysis, they show that the revenue depends only on the disclosure rule, but not on the auction format. Their analysis further shows that whether more or less information shall be disclosed depends on the number of bidders and whether bidders signaling concern is increasing or decreasing. Our analysis challenges these findings by showing that it is not the number of bidders that matters, but how disclosure affects the signaling value of non-participation. Various devices could implement such a disclosure policy: revealing all bids, revealing a list of bidders who placed a bid, or requiring an entry fee. By allowing a strict subset of bidders to distinguish themselves from worse types, the auctioneer ensures the worst possible beliefs about non-participating bidders. This maximum penalty for non-participation, in terms of expected inferences, leads to the extraction of the maximum signaling value from each bidder, thereby maximizing the auction's expected revenue. In an extension we add to this by showing that also the shape of the signaling function matters, but not its slope.[8] Bos and Truyts compare second-price and English auctions when bidders have *linear* signaling concern. They assume that only the winner's identity and her own payment get revealed. In the light of our results, the difference in auction revenue stems from the difference in information revealed about a bidder's participation.

Our paper is also related to the literature on mechanism design with aftermarkets. Calzolari and Pavan (2006a,b) study contracting environments where the agent participates in an aftermarket. They find conditions under which no information release to the aftermarket is optimal. Dworczak (2020) analyzes an auction environment with a very general aftermarket. He restricts the analysis to cut-off mechanisms in which the information revealed about the winner only depends on the losers' bids. These mechanisms rule out disclosure of information contained only in the winner's bid, such as the price in a first-price auction or the entire vector of payments in the all-pay auction, which we show is optimal in some cases. Also Molnar and Virag (2008) study auctions with an aftermarket, where only the winner has a signaling concern (and, consequently, there is no aftermarket if there is no winner). They show that it is optimal to reveal (conceal) the winner's type when the signaling incentive is convex (concave). In our setting all bidders care about how they are perceived, irrespective of whether they win. This brings about a novel decomposition of revenue into the standard non-signaling component and a signaling component, which can be analyzed using methods from information design. In addition, a new trade off arises from the bidders' endogenous outside option, implicitly given by the signaling value from abstention, which yields new insights in the concave and convex cases.

Information disclosure in auctions has first been analyzed in the setting of affiliated values by Milgrom and Weber (1982). Mechanism design problems with allocative and informational externalities have also been studied by Jehiel and Moldovanu (2000, 2001). The underlying assumption in this strand of literature is that an agent's valuation depends also on other agents' private information (and allocation). In our setting a bidder's utility is affected by the aftermarket's belief about her own valuation, while such beliefs have no impact in the literature on mechanism design with interdependent valuations.

The paper is organized as follows. Section 2 introduces the formal setting. Section 3 provides our main result, a detailed analysis for the case of linear signaling concerns. We restore revenue equivalence and derive optimal auctions. Section 4 studies extensions of our main model to cover general mechanisms and non-linear signaling concerns. We derive optimal auctions when the signaling concerns are convex, or concave, and uncover a novel trade-off in the latter case. Section 5 concludes.

2. Formal setting

We consider n bidders, who bid for a single object in an auction, and also care about the inference of an outside observer about their type.

Bidder i's valuation for the object (her 'type'), is denoted V_i, and is assumed i.i.d. and drawn according to a distribution function F with support on $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$. Let $f \equiv F'$ denote the density function, $G \equiv F^{n-1}$ the distribution function of the highest order statistic among $n-1$ remaining valuations and $g \equiv G'$ the corresponding density function. Bidder i's realization of V_i, denoted v_i, is her private information, but the number of bidders and the distribution F are common knowledge. The auctioneer's value for the object is zero.

We consider *standard auctions* in which each bidder submits a (non-negative) bid b_i, the bidder who places the highest bid (ties broken at random) wins, and bidder i's payment p_i depends on the entire vector of bids, i.e., $p_i(b_1, \dots, b_n)$. Implicitly we allow for reserve prices or entry fees that affect bidders' participation decisions. Abstention we formally model as 'bidding' $b_i = \emptyset$. Hence, the bid space is $B := [0, \infty) \cup \{\emptyset\}$. A bidder who abstains does not make a payment and never wins the object (also if all other bidders abstain as well).

[7] There are also contributions about information transmission comparing specific auction formats followed by oligopoly competition. See, e.g., Goeree (2003), Das Varma (2003), Katzman and Rhodes-Kropf (2008) and von Scarpatetti and Wasser (2010).

[8] More precisely, the slope only matters insofar that steeply decreasing signaling functions impede existence of equilibria. This has been pointed out already by Giovannoni and Makris (2014).

After the auction was conducted, the winning bidder's identity is publicly revealed.[9] In addition, the auctioneer chooses an *information disclosure policy* revealing information about the submitted bids. Formally, an information disclosure policy (σ, S) consists of a signal space S and a mapping $\sigma : B^n \to \Delta S$. Upon submitting bids $b_1, \ldots, b_n$ a signal $s \in S$ is drawn (potentially at random) from the distribution $\sigma(b_1, \ldots, b_n) \in \Delta S$. This formulation encompasses as extremes (i) a full revelation policy, where $S = B^n$ and $\sigma(b_1, \ldots, b_n) = \mathbb{I}_{(b_1,\ldots,b_n)}$, and (ii) a no revelation policy, where, e.g., $S = \{s\}$ is a singleton.[10] Many real-world examples of auctions do not feature explicit bid revelation, but implicit revelation via observable prices. For example, guidelines on public procurement demand that the final price is public.[11] Our model captures this as follows: for any standard auction let $S = \mathbb{R}^n_+$, and $\sigma(b_1, \ldots, b_n) = \mathbb{I}_{\left\{(p_1(b_1,\ldots,b_n),\ldots,p_n(b_1,\ldots,b_n))\right\}}$. That is, the auctioneer reveals each bidders' payment, and thus implicitly some information about submitted bids.[12] We denote $\mathcal{O} := (i^\star, s)$ the public outcome of the auction, where $i^\star$ is the winner's identity and s the signal revealed by the auctioneer.

Each bidder cares about winning the object, and about the inference of an outside observer, the 'receiver', about her type. This receiver can represent, e.g., the general public or press, business contacts or acquaintances of the bidder, or experts related to the object at sale. The receiver observes the auction outcome $\mathcal{O}$, but no further information about the bidding process, and forms a posterior belief about each bidder's type, denoted as $\mu_i(\mathcal{O})$. We assume that a bidder's utility depends *linearly* on the posterior mean of the receiver's belief about the bidder's type.[13] A bidder's utility is given by

$$u_i(v_i, \mathcal{O}) = \begin{cases} v_i - p_i + \lambda \mathbb{E}\big(V_i|\mathcal{O}\big), & \text{if } i = i^\star, \\ -p_i + \lambda \mathbb{E}\big(V_i|\mathcal{O}\big), & \text{if } i \neq i^\star. \end{cases} \tag{1}$$

The parameter $\lambda \geq 0$ measures the strength of the bidders' signaling concern. This signaling function represents a reduced form of a (continuation) game in which the receiver chooses an action that directly affects the bidder's payoff. Note that a bidder's utility is not affected by the receiver's belief about other bidders' types. For instance, from an individual bidder's perspective it is equivalent to have either a different or the same receiver for each bidder.

There are a few implicit assumptions that warrant some discussion. First, the bidders' signaling value only depends on the posterior mean of her type. In our setting with continuous types this assumption ensures tractability, for otherwise the mere information design problem becomes intractable (see also Dworczak and Martini (2019)). Second, the signaling value does not depend on the bidder's true type. Adding a type-dependency renders the bidder's essentially non-linear in type and thereby hampers the auction design part of our analysis. Third, we implicitly assume a bidder would like to be perceived as 'high' type. This is a realistic assumption in many contexts, see the examples in the introduction and below. Note that if bidders prefer being perceived as low types there are both incentives for bidding high (to win the object) and low (to get higher signaling value). Depending on the weighting of these two forces in the bidders' utility function we may encounter countervailing incentives as in Lewis and Sappington (1989), impeding the existence of separating equilibria in the first place.

Any standard auction with information disclosure defines a signaling game among bidders and the receiver. We consider symmetric perfect Bayesian equilibrium, consisting of the bidders' bidding strategies $\beta : [\underline{v}, \overline{v}] \to B$ and the receiver's belief $(\mu_1, \ldots, \mu_n)$. Each bidder's bidding strategy is optimal, given the other bidders' bidding and the receiver's beliefs. Also, the receiver's beliefs are Bayesian consistent with the bidding strategy. In our analysis we focus on equilibria in which (participating) bidders use strictly increasing bidding functions. Restricting to symmetric equilibria follows the usual practice in the literature, given the symmetric environment that we study. Only symmetric and strictly increasing equilibria implement an efficient allocation of the object to the bidder with the highest valuation. Besides this efficiency-driven motivation, it is possible to show that if the bidders' signaling concern is not too strong (compared the payoff from winning) all equilibria of a standard auction have to be in increasing strategies. It thus allows for a straightforward comparison with the no-signaling benchmark.

We conclude this section with two applications of our formal setting.

Takeover bidding with career concerns. Suppose the bidders are the managers of firms, bidding in a takeover. The profit of the merged firm, if manager i's firm wins, is v_i. We assume that v_i is a proxy for the manager's ability, i.e., it is the manager who makes the merger profitable, and the efficient takeover involves the most talented manager. Managers maximize their firm's profit, i.e., conditional on winning the difference between post-merger profit v_i and takeover price p. This is reasonable, for instance if the manager's compensation bases on the firm's profit. After the takeover auction concluded, a competitive labor market opens for the

---

[9] Revealing the winner's identity constitutes more a refinement than an assumption. Indeed, when no identities and no information on bids gets revealed, there is no updating of the public's belief. But then the winner has an incentive to disclose her identity, because this reveals her valuation to be higher than average. Given this incentive, it is not sustainable at equilibrium for the winner to remain anonymous, as they will eventually disclose their identity. Designers cannot prevent individuals from revealing their identities if they choose to do so, because the winner becomes the owner of the object at sale and cannot be legally prevented from using it.

[10] Technically there are many alternative variants for specifying these policies. E.g., no revelation can be obtained via an arbitrary signal space and a degenerate distribution σ which reveals the same signal, irrespective of the actual bids.

[11] See also Footnote 6.

[12] For instance, in a first-price auction the revelation of the winner's payment essentially reveals the winner's bid. In a second-price auction, the revelation of the winner's payment reveals that (i) the winner's bid was weakly higher, (ii) some losing bidder placed exactly this bid, (iii) all losing bidders placed a weakly lower bid. In an all-pay auction revelation of all payments is equivalent to revelation of all bids.

[13] Note that we do not assume that a bidder's type v_i *directly* affects the receiver's payoff. The receiver may care about some other characteristic of the bidder, which is correlated with the bidder's type. See also the examples at the end of this section.

managers' services. As in the models of career concerns (e.g., Holmström (1999)), the wage to manager i in this competitive market with risk-neutral firms equals the manager's expected ability, i.e., we have $w_i^\star = \mathbb{E}(V_i|\mathcal{O})$.

Image concerns in a charity auction. Suppose a good with common value $\Gamma > 0$ is sold in a charity auction. Bidders derive value from obtaining the good and from contributing to the charity. Paying p dollars to the charity yields utility νp, where $\nu < 1$. Hence, obtaining the good while paying a price p yields $\Gamma - p + \nu p$, while only paying p without obtaining the good yields $-p + \nu p$. Transforming appropriately yields the bidder's utility is $v\Gamma - p$, and $-p$ respectively (i.e., multiplying through by $v = 1/(1-\nu)$). In addition to the described utility from direct participation in the charity, bidders have preferences for their social image. Following Bénabou and Tirole (2006), each bidder's utility also depends additively on the expected altruism, i.e., on societies expected value of v (resp., ν). Combining the bidder's direct utility from participating in the auction with the utility from her social image yields the utility as given by Equation (1).

3. Analysis

For the first part of the analysis we fix some standard auction cum disclosure policy A. As mentioned earlier, we focus on symmetric and strictly increasing bidding. Denote $\tau \in [0,1)$ the participation threshold, i.e. a bidder participates if and only if her valuation exceeds τ. Define $Rev^{\mathcal{M}}(\tau)$ the revenue of auction A if bidders have no signaling concern, i.e.,

$$Rev^{\mathcal{M}}(\tau) := n \int_{\tau}^{\overline{v}} \left(G(\tau)\tau + \int_{\tau}^{v} g(x)x\mathrm{d}x \right) \mathrm{d}F(v),$$

and $\mathcal{W}^A_{\tau,\emptyset}$ denote the (interim) expected signaling value of a bidder who abstains from participation.

Proposition 1 *(Revenue decomposition). Consider a standard auction A, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. The revenue in this auction is given by*

$$Rev^{\mathcal{M}}(\tau) + n\left(\lambda \mathbb{E}(V) - \mathcal{W}^A_{\tau,\emptyset}\right). \tag{2}$$

Proof. We expand standard arguments from auction theory to our setting with signaling bidders (Riley and Samuelson, 1981; Krishna, 2009). Denote $m^A_\tau(v)$ the expected payment and $\mathcal{W}^A_\tau(v)$ the expected value from signaling of a bidder with valuation v. By assumption, we have $m^A_\tau(v) \equiv 0$ and $\mathcal{W}^A_\tau(v) \equiv \mathcal{W}^A_{\tau,\emptyset}$ for all $v < \tau$. Now consider a bidder with valuation $v \geq \tau$. His expected payoff from mimicking type $\tilde{v} \geq \tau$ is

$$\Pi(v, \tilde{v}) = G(\tilde{v})v - m^A(\tilde{v}) + \mathcal{W}^A_\tau(\tilde{v}). \tag{3}$$

At equilibrium the bidder's payoff is $\Pi(v) := \Pi(v, v)$ and using the envelope theorem[14] it follows that

$$G(v)v - m^A(v) + \mathcal{W}^A_\tau(v) = \Pi(v) = \Pi(\tau) + \int_{\tau}^{v} G(x)\,\mathrm{d}x. \tag{4}$$

A bidder with valuation τ is indifferent whether to participate, if

$$\Pi(\tau) = \mathcal{W}^A_{\tau,\emptyset}. \tag{5}$$

Using (4) and (5) we express the *interim* expected payment of a bidder as follows

$$m^A(v) = G(v)v - \int_{\tau}^{v} G(x)\,\mathrm{d}x + \mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset} = G(\tau)\tau + \int_{\tau}^{v} g(x)x\,\mathrm{d}x + \mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}, \tag{6}$$

where the second equality uses integration by parts. With this we can write the auctioneer's revenue as

$$Rev^A(\tau) = n\int_{\underline{v}}^{\overline{v}} m^A(v)\mathrm{d}F(v) = n\int_{\tau}^{\overline{v}} m^A(v)\mathrm{d}F(v)$$

$$= Rev^{\mathcal{M}}(\tau) + n\int_{\tau}^{\overline{v}} \left(\mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}\right)\mathrm{d}F(v)$$

[14] See Milgrom and Segal (2002). The objective in (3) is differentiable in v, and its derivative $G(\tilde{v})$ is uniformly bounded.

$$= Rev^{\mathcal{M}}(\tau) + n\left(F(\tau)\mathcal{W}^A_{\tau,\emptyset} + \int_{\tau}^{\overline{v}} \mathcal{W}^A_{\tau}(v)\mathrm{d}F(v) - \mathcal{W}^A_{\tau,\emptyset} \right)$$

$$= Rev^{\mathcal{M}}(\tau) + n\left(\lambda\mathbb{E}(V) - \mathcal{W}^A_{\tau,\emptyset}\right).$$

The last equality uses linearity of the bidders' signaling concern and the law of iterated expectation. □

The Proposition shows that revenue in the auction with signaling bidders can be decomposed into the revenue of the respective auction without signaling concern, and the revenue that arises solely from the bidders' signaling concern. Standard auction theory shows that the former is independent of the details of the auction, as long as it facilitates an equilibrium in strictly increasing bidding strategies.

It is immediate from Proposition 1 that differences in auction revenue are solely due to the signaling value to non-participating bidders $\mathcal{W}^A_{\tau,\emptyset}$. Varying disclosure policies lead to a mere redistribution of total signaling value across bidder types. The auctioneer extracts signaling value only from participating bidders (irrespective of the re-distribution), where the signaling value from not participating serves as an endogenous outside option.

Using the disclosure policy (σ, S) to shift signaling value to types $v \geq \tau$ therefore increases revenue for two reasons: (i) it increases the signaling value of types from which this value can be extracted, and (ii) it lowers the bidders' outside option and allows for extracting *more* of their signaling value. Consequently, revealing whether a bidder participated maximizes the auctioneer's revenue.

Proposition 2 *(Optimal disclosure). Consider a standard auction in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. For any disclosure policy, the associated revenue is bounded by*

$$Rev^{\mathcal{M}}(\tau) + n\lambda\big(\mathbb{E}(V) - \mathbb{E}(V|V<\tau)\big). \tag{7}$$

Moreover, the bound is attained for any disclosure policy that reveals whether a bidder participated.

Proof. We have $\mathcal{W}^A_{\tau,\emptyset} \geq \lambda\mathbb{E}(V|V<\tau)$, because by assumption all types below τ do not participate, and are hence lumped together. Plugging this into (2) yields the bound in (7). Upon disclosing whether a bidder participated it is straightforward that the latter inequality turns into an equality. Hence, the revenue bound is tight. □

Remark 1. It is not important which information on submitted bids gets disclosed. The only important piece of information that are the individual bidders' participation decisions. While the former has redistributive effects, i.e., how signaling value is distributed across types, the latter directly affects the auctioneers revenue, because determines the amount of signaling value the auctioneer can extract from participating bidders.

There are several practical ways for implementing the optimal information disclosure. First, disclosing all bids necessarily reveals whether an individual bidder actually placed a bid. For example, regulations in takeover bidding demand all bids to be public. Second, the auctioneer discloses a list of bidders that submitted a valid bid, i.e., a bid above the reserve price. Third, the auctioneer charges an *entry fee* and reveals each bidder's final payment. This way, the bidding process remains confidential, but participation decisions become public. In particular, the latter two variants of information disclosure do not necessarily reveal the entire ranking of bidders.

We next briefly turn to optimal levels of participation, comparing auctions with and without signaling bidders. We focus on auctions with optimal information disclosure, i.e., auctions which make participation decisions public, as these attain the maximal revenue determined in Proposition 2. Denote with $\tau^\star(\lambda)$ the optimal level of participation under signaling strength λ.[15] Note that $\tau^\star(0) = \tau^{\mathcal{M}}$, where $\tau^{\mathcal{M}}$ denotes the optimal participation threshold in an auction without signaling concerns. The corollary shows that the optimal level of participation (weakly) increases in the signaling strength and there exists a finite threshold of signaling strength which makes full participation optimal.

Corollary 1 *(Optimal Participation). Assume virtual valuations are increasing. We have that*

(i) $\tau^\star(\lambda) \leq \tau^\star(\lambda') < \tau^{\mathcal{M}}$ *for all* $\lambda > \lambda' > 0$.
(ii) There exists $\overline{\lambda}$ *such that* $\tau^\star(\lambda) = \underline{v}$, *for all* $\lambda > \overline{\lambda}$.

Proof. From Riley and Samuelson (1981) we know that

$$\left(Rev^{\mathcal{M}}\right)'(\tau) = nF^{n-1}(\tau)\big(1 - F(\tau) - \tau f(\tau)\big) = -nf(\tau)F^{n-1}(\tau)\left(\tau - \frac{1-F(\tau)}{f(\tau)}\right).$$

[15] In general there may not be a unique optimal level of participation. Our assumption of increasing virtual valuations in Corollary 1 guarantees both existence and uniqueness of $\tau^\star(\lambda)$.

Provided that virtual valuations $v - \frac{1-F(v)}{f(v)}$ are strictly monotone we have that $Rev^{\mathcal{M}}(\tau)$ has a unique maximum $\tau^{\mathcal{M}}$. Furthermore, $\left(Rev^{\mathcal{M}}\right)'(\tau) < 0$ for all $\tau > \tau^{\mathcal{M}}$, and $\left(Rev^{\mathcal{M}}\right)'(\tau) > 0$ for all $\tau \in (\underline{v}, \tau^{\mathcal{M}})$. Together with the observation that $\mathbb{E}(V|V < \tau)$ strictly increases in τ, (i) follows. To prove (ii) note that the derivative of $Rev^{\mathcal{M}}(\tau)$ is bounded. Hence, as soon as λ becomes sufficiently large we have that $\lambda \frac{\partial}{\partial \tau}\mathbb{E}(V|V < \tau) > \left(Rev^{\mathcal{M}}\right)'(\tau)$ for all $\tau > \underline{v}$ and thus full participation maximizes revenue in the auction with signaling concerns. □

4. Discussion

This section studies extensions of our model and discusses equilibrium selection, other mechanisms, and (non) robustness of the results about linear signaling concerns.

4.1. Equilibrium selection

Our focus in the analysis so far has been on equilibria with strictly increasing bids. These equilibria are the natural focus in auction theory, because they yield the efficient allocation (net of not allocating the object when all valuations are low). In this section we extend the analysis to all equilibrium outcomes that may arise in an auction with information disclosure. First, using standard arguments we show that equilibrium bidding has to be weakly increasing.

Lemma 1. *Fix some auction with information disclosure. Every symmetric equilibrium is in weakly increasing strategies.*

Proof. Fix types $v < v'$ and suppose b (b') is an equilibrium bid of type v (v'). Denote $H(b)$ the winning probability, $m(b)$ the expected payment, and $\mathcal{W}(b)$ the expected signaling value from bidding b. Equilibrium conditions yield

$$vH(b) - m(b) - \mathcal{W}(b) \geq vH(b') - m(b') - \mathcal{W}(b'),$$

and

$$v'H(b') - m(b') - \mathcal{W}(b') \geq v'H(b) - m(b) - \mathcal{W}(b).$$

Combining the two inequalities, we get $(v' - v)\big(H(b') - H(b)\big) \geq 0$, thus $H(b') \geq H(b)$, and therefore $b' \geq b$. □

Following the Lemma, every equilibrium of an auction with information disclosure gives rise to an interval of types $[0, \tau)$ that do not participate. In addition, the implemented allocation is weakly monotone, perhaps with bunching of (intervals of) adjacent types. But then the revenue formula obtained in Proposition 1 changes only in its first component. Most importantly, it remains optimal to reveal individual participation decisions. While this answers the questions of optimal information disclosure, it leaves open the question of the optimal auction design. Using insights from Myerson (1981) bunching is never optimal if the distribution F satisfies a regularity condition. In that case any auction that gives rise to an equilibrium with strictly increasing bid functions together with an optimal information disclosure yields the maximal revenue for the auctioneer.

4.2. General mechanisms

When bidders have no signaling concern the restriction to auctions is known to be without loss if Myerson's regularity condition applies. In principle, a similar reasoning holds in our setting. Due to additive separability of the bidders' utility, the revenue can be decomposed into a component capturing the revenue absent any signaling concern, and a component capturing additional rents extracted due to the bidders' signaling concern. Regarding the first component it is then straightforward that under Myerson's regularity assumption implementing an allocation rule that allocates the object to the bidder with the highest valuation, if that valuation exceeds a critical threshold, is optimal. Maximizing the signaling value given this allocation is then the same exercise as in the main body of this manuscript. Hence, under the regularity assumption our focus is not restrictive.

What questions this reasoning is the following observation. Unlike in classical mechanism design, the bidders' outside option here is affected by information disclosure. As an illustration, consider the following two-stage all-pay auction with an entry fee. In the first stage, a bidder can enter the auction by paying a fee f, where this payment is made public. The second stage amounts to an all-pay auction with reserve price r among bidders who entered at stage one. I.e., a bidder who paid the entry fee submits a bid b. The highest bid – if it exceeds the reserve price r – wins the object. The final payment of *each* active bidder is $b + f$, i.e., the payment rule at the auction stage is all-pay. All other bidders make no payment at all.

It can be established that by setting $f = \mathbb{E}(V|V < \tau) - \underline{v}$, i.e., equal to the difference between the signaling value of being perceived as below type τ and being perceived as the lowest type, together with an appropriately chosen reserve price the auction exhibits the following equilibrium. All bidder types pay the entry fee and bidders with a type above τ place a bid. Moreover, this equilibrium yields the auctioneer a revenue equal to the sum of the auction's stand-alone revenue $Rev^{\mathcal{M}}(\tau)$ and the entire signaling value of bidders $n\lambda\mathbb{E}(V)$. In other words, the auctioneer extracts all the signaling value by charging an upfront fee. Importantly, this equilibrium builds on the belief that a bidder who does not pay the entry fee is of the lowest type $\underline{v}$.

The mechanism in this example faces two major criticisms. First, the equilibrium that yields the desired payoff is not robust. It rests on the particular off-path belief stated above. It can be shown, that this equilibrium does not survive the D1-refinement. In general, this brings upon a tricky question for mechanism design with signaling concerns. Also in the model at hand it is without loss of generality to assume all bidders participate in the mechanism, because the mechanism can be extended to give non-participating bidders the same payoff as from abstention. Hence, non-participation is an off-path event. Without signaling motives this is innocuous, but with signaling concerns the choice of beliefs crucially affects the value of the outside option. To increase revenue, the seller would like this outside option to be as unattractive as possible. But from a game-theoretic viewpoint we also wish equilibrium play to be reasonable. Hence, we may demand that the choice of an off-path belief following non-participation is somewhat reasonable, i.e., the equilibrium is not just a perfect Bayesian equilibrium, but also survives some given refinement. The major challenge is then to formalize this refinement such that the associated constraint maintains tractability.

A second criticism stems from envisioning the above mechanism in the real world. Bidders with valuation below τ make a strictly positive payment, without planning on actually bidding for the object (because the reserve price is too high). Their only motive for paying the seller is to avoid the before-mentioned off-path belief. We call this *belief extortion*: by setting the most pessimistiv off-path belief, the seller forces bidders to at least enter the auction. Such practices seem highly unrealistic in practice. A technical way around this could be a formalization via an additional requirement of the mechanism. Such as, a bidder only makes a positive payment if she acquires a strictly positive chance of getting the object. While this seems straightforward, it becomes tricky when we consider random mechanisms. The designer may give an arbitrarily small probability of winning and we can look at mechanisms that have this probability converge to zero. In the limit we obtain the same outcome as in the example. Hence, such small probability of winning mechanisms face the same criticism as the mechanism in our example.

4.3. Non-linear signaling concerns

In this section, we briefly consider non-linear signaling concerns. As before we assume a bidder's utility depends on the posterior mean $\mathbb{E}(V_i|\mathcal{O})$, but consider increasing signaling functions $\Phi(\mathbb{E}(V_i|\mathcal{O}))$ that need not be linear. An auction with information disclosure induces in equilibrium a distribution over posterior means. We denote by H_τ^A this distribution, in an equilibrium where bidders with valuation above τ participate and use a strictly increasing bidding strategy. Using the same arguments as for proving Proposition 1 we get the following Lemma.[16]

Lemma 2. *Consider a standard auction A, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. The revenue in this auction is given by*

$$Rev^{\mathcal{M}}(\tau) + n\left(\int_{\underline{v}}^{\overline{v}} \Phi(\tilde{v})\mathrm{d}H_\tau^A(\tilde{v}) - \mathcal{W}_{\tau,\emptyset}^A\right). \tag{8}$$

With non-linear signaling concerns the total signal value $\int_{\underline{v}}^{\overline{v}} \Phi(\tilde{v})\mathrm{d}H_\tau^A(\tilde{v})$ crucially depends on the shape of Φ. Upon considering only this term in the auctioneer's objective, her problem is that of finding the distribution over posterior means maximizing the signaling value. In doing so, the auctioneer is constrained by (i) having to reveal the winner's identity, and (ii) being able to reveal only information that she gathered from the bidders. Following insights from information design (Dworczak and Martini, 2019) the posterior expectations are a mean-preserving contraction of the prior distribution F, that can be implemented via some disclosure rule.[17] Yet the disclosure policy not only affects the total signaling value, but also the endogenous outside option $\mathcal{W}_{\tau,\emptyset}$. In general, what turns out optimal regarding one of these components may not be optimal regarding the other. We illustrate this by considering two special cases of Φ for which the information design problem sketched above has a particularly simple solution.

Convex signaling concern. When Φ is convex, the signaling value is maximal upon disclosing all available information, i.e., inducing the distribution over posterior means $H_\tau^{\max}$. This is achieved by revealing all bids, which indirectly reveals bidders' types due to the strictly increasing bid function. Coincidentally, such an information disclosure policy also minimizes the outside option $\mathcal{W}_{\tau,\emptyset}$ as already discussed in the previous section. Hence, the optimal information disclosure amounts to disclosing all bids. Upon doing so, the specific payment rule does not further affect the auctioneer's revenue, because the first term in the revenue does not depend on it.

Proposition 3 *(Optimal auction under convexity). Consider a standard auction, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. With convex signaling concern, the revenue in this auction is at most*

$$Rev^{\mathcal{M}}(\tau) + n\left(\int_{\tau}^{\overline{v}} \Phi(v)dF(v) - (1-F(\tau))\Phi(\mathbb{E}[V|V \le \tau])\right). \tag{9}$$

[16] The proof is available in Bos and Pollrich (2023).

[17] Additional details are available in Bos and Pollrich (2023).

Any standard auction with full revelation of all bids exhibits the described equilibrium and attains the revenue bound.

Proof. For every distribution over means H that can be implemented we know that it $H_\tau^{\max}$ has to be mean-preserving spread of it. Convexity of Φ then implies that,

$$\int_{\underline{v}}^{\overline{v}} \Phi(v) d H(v) \leq \int_{\underline{v}}^{\overline{v}} \Phi(v) d H_\tau^{\max}(v) = F(\tau)\Phi(\mathbb{E}[V|V \leq \tau]) + \int_{\tau}^{\overline{v}} \Phi(v) d F(v). \tag{10}$$

Together with our previous observation that $\mathcal{W}_{\tau,\emptyset}^A \geq \Phi(\mathbb{E}[V|V \leq \tau])$, with equality upon disclosing all bids, the revenue bound (9) follows.

To show the revenue bound (9) can be attained, consider a standard auction with a full disclosure policy, i.e., where all bids are disclosed. Denote $\beta^{\mathcal{M}}(v)$ the respective equilibrium bid in the auction without signaling concern, and $m^{\mathcal{M}}(v)$ the respective expected payment upon bidding as type v, i.e., placing the bid $\beta^{\mathcal{M}}(v)$. Now define $\beta(v) = \beta^{\mathcal{M}}(v) + x(v)$ such that for the expected payment in a separating equilibrium of the auction with signaling we have $m(v) = m^{\mathcal{M}}(v) + \Phi(v)$. By definition, β is strictly increasing. Hence, revealing the bid in fact reveals the type. It remains to show that bidding according to β indeed forms an equilibrium. Specify the system of beliefs such that (i) a bidder who did not participate (i.e., bids $b = \emptyset$) has expected type $\mathbb{E}(V|V \leq \tau)$ (formally, the bidder's type is distributed according to the truncation of F on the interval $[\underline{v}, \tau]$), (ii) upon observing bid b the receiver beliefs to face type $v = \beta^{-1}(b)$ (if such a type exists) and (iii) for all other bids the receiver believes the bidder's type is $\underline{v}$. Note that the expected payoff of type v who mimicks type v' is

$$G(v')v - m(v') + \Phi(v') = G(v')v - m^{\mathcal{M}}(v')$$

As $\beta^{\mathcal{M}}$ was an equilibrium of the auction without signaling, the above objective is maximized upon bidding $\beta(v)$, i.e., bidding as the true type. We have thus verified that the bidding strategy β together with the specified system of beliefs forms an equilibrium. □

Proposition 3 implies revenue equivalence with the additional requirement that the auction is augmented by an optimal disclosure policy that reveals all submitted bids. This finding has a couple of implications relevant in practice. First, without an optimal disclosure policy revenue is typically strictly lower than the bound given in Equation (9). An example where an optimal disclosure policy is used is takeover bidding, where regulations require all bidding to be public. Our findings imply, that the specific payment rule has no further impact on revenue. Second, with a given non-optimal disclosure policy revenue equivalence does not obtain. Take for instance a first- and a second-price auction that reveal the winner's payment. Inference on winning and losing bidders' types is different, hence the signaling values differ. As a consequence, when the choice of a disclosure policy is restricted, auction design is critical. For instance, regulations for public procurement require disclosure of the final price but do not allow any further revelation of bids. When bidders have non-linear signaling concerns, the choice of an auction format then crucially affects revenue.

Concave Signaling Concerns. With concave signaling concerns the total signaling value is maximal upon disclosing the least amount of information, i.e., only the winner's identity. Such an information disclosure yields a high outside option $\mathcal{W}_{\tau,\emptyset}$ and is typically not revenue maximizing. The auctioneer faces a trad-off between increasing the total signaling value and decreasing the bidders' outside option. This trade-off is also affected by the induced participation threshold τ.

With full participation, i.e., $\tau = \underline{v}$, every bidder submits a bid in equilibrium. Hence, disclosing whether a bidder submitted a bid does not provide additional information on the bidders' valuations. But it allows for reducing the bidders' outside option, because abstention will then be *punished* by the most pessimistic belief. Hence, disclosing only the winner's identity together with information on bidders who actually submitted a bid yields maximal revenue. The following Proposition shows that this continues to hold if the participation threshold is sufficiently low.

Before we state the Proposition, let us define H_τ^P the distribution over the posterior expectations that is a mean-preserving contraction of the prior distribution F, which arises from a disclosure policy which reveals the winner's identity and whether a bidder participated in an auction when the participation threshold is $\tau \in [\underline{v}, \overline{v}]$. Therefore, we denote $H_{\underline{v}}^P$ the corresponding mean-preserving spread of the *minimum information* induced by the auction at the equilibrium. Moreover, we define

$$Rev^P(\tau) = Rev^{\mathcal{M}}(\tau) + n\Big(\int_{\underline{v}}^{\overline{v}} \Phi(v) \mathrm{d} H_\tau^P(v) - \mathbb{E}[V|V < \tau] \Big). \tag{11}$$

Proposition 4. *Consider a standard auction, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy.*

(i) *If participation is fully observable we have that $Rev(\tau) \leq Rev^P(\tau)$.*
(ii) *There exists $\tau' > \underline{v}$ such that $Rev(\tau) \leq Rev^P(\tau)$ whenever $\tau < \tau'$.*

Proof. From Proposition 1 we have that revenue equals (2). With observable participation we have $\mathcal{W}^A_{\tau,\emptyset} = \mathbb{E}[V|V < \tau]$, independent of the specific auction format. Furthermore, because Φ is concave the signaling value $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_\tau(v)$ is maximal when only the winner's identity is disclosed in addition, i.e., when $H^A_\tau = H^P_\tau$. This proves (i).

To prove (ii), note that for every auction in which participation is not fully observable we have $\mathcal{W}^A_{\tau,\emptyset} > \mathbb{E}[V|V < \tau]$. Moreover, for $\tau = \underline{v}$ we have $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_{\underline{v}}(v) \le \int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^{\min}(v)$ by concavity of Φ. Hence, $Rev(\underline{v}) < Rev^P(\underline{v})$. Both $\mathcal{W}^A_{\tau,\emptyset}$ and $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_\tau(v)$ are continuous in the participation threshold τ, hence (ii) follows by continuity from the previous assertion. □

Proposition 4 derives an upper bound for the revenue if either participation is fully observable, or many bidder types participate. In other cases, i.e., when few bidder types participate in the auction, other information disclosure policies yield higher revenue. In order to increase the signaling value, some participation decisions have to be concealed, which then increases the bidders' outside option.

Proposition 4 reveals a fundamental difference between pure information design and mechanism design with information disclosure. In information design the sender has costless access to all information, while in mechanism design the information is privately held by the agents and has to be elicited from them. In our setting the auctioneer benefits from revealing additional information, namely whether a bidder participated, in order to reduce the bidders' outside option and increase her revenue. Note that in equilibrium this additional disclosure does not provide additional information to the public, as it only affects the beliefs following actions not taken on the equilibrium path.

The bound derived in Proposition 4 stems from a disclosure policy that reveals only the winner's identity, and a list of participating bidders. In particular, revealing additional information such as the winner's payment in first- or second-price auctions reduces revenue. In some contexts it is mandatory to disclose such payments. Does this necessarily imply that revenue falls drastically? We show that in theory there is a remedy to eliminate the adverse effects of information leakage via observable payments.

Proposition 5. *For every $\varepsilon > 0$ and every τ there is an auction with mandatory disclosure of all payments that exhibits an equilibrium with strictly increasing bidding strategies, for which $Rev(\tau) > Rev^P(\tau) - \varepsilon$.*

Proof. See the appendix. □

Remark 2. With concave signaling concerns an auction does not necessarily constitute an optimal mechanism, hence our focus on equilibria in strictly increasing strategies is restrictive. To see this let us consider the following example with 2 bidders, valuations drawn from a uniform distribution on $[1, 2]$, and a signaling function $\Phi(v) = k\sqrt{v}$ with $k > 0$. Therefore, the optimal auction uses no reserve price, and participation is fully observable. Following Propositions 4 and 5, the maximal revenue of an auction is then

$$Rev^A = \frac{4}{3} + 2k\left(\frac{1}{2}\sqrt{\frac{5}{3}} + \frac{1}{2}\sqrt{\frac{4}{3}}\right).$$

Now consider a lottery, in which the object is allocated at random. Provided that both bidders participate, type v's expected utility is $\frac{1}{2}v + k\sqrt{\frac{1}{2}}$. Note that the allocation, i.e., who is assigned the object, does not reveal new information about the bidders' types. To ensure full participation the seller can charge a participation fee of $\frac{1}{2} + k\sqrt{\frac{3}{2}}$, thus revenue is

$$Rev^P = 1 + 2k\sqrt{\frac{3}{2}}.$$

Clearly, for sufficiently large k we have $Rev^P > Rev^A$.[18] Recall that we assume the final allocation (i.e., who gets the object) is observable. Therefore, a mechanism that implements an allocation rule that conditions on reported types necessarily reveals information about bidders. With a concave signaling function, information revealed via the allocation reduces the signaling value the auctioneer can extract. If signaling concerns are sufficiently strong, the gain in signaling value outweighs the loss in terms of standard revenue $Rev^{\mathcal{M}}$, hence an auction is no longer optimal. Whether a lottery represents an optimal mechanism depends on the marginal gain from improving the allocation versus the marginal loss in terms of signaling value this brings about.[19]

5. Conclusion

In this paper, we analyze optimal auctions in an independent private values environment with signaling, i.e. where bidders care about the perception of a third party. To keep the analysis concise and tractable we focused on linear, convex and concave signaling

[18] Straightforward computations lead to a threshold of $\hat{k} \approx 87.84$. This threshold would reduce with either more bidders, more concave signaling function Φ, or a more spread-out distribution.

[19] However, note that a formal analysis is all but straightforward. The presence of signaling prevents us from using standard methods, such as pointwise maximization of the objective.

concerns. The results of Dworczak and Martini (2019) indicate that the disclosure policy maximizing the signaling value for general preferences is a combination of intervals where the type is fully disclosed and intervals on which types are fully pooled. However, it is not straightforward to translate such a disclosure rule into a payment rule for a standard auction. Understanding the polar cases of convexity and concavity allows us to address a preference for the aftermarket that has been studied in the literature on information design, namely where Φ is a distribution function.[20] Under regularity conditions, there is a unique value $\hat{v}$ such that Φ is convex on $[\underline{v}, \hat{v}]$ and concave on $[\hat{v}, \overline{v}]$. Hence, if the participation threshold is sufficiently high we are back in the concave case. Otherwise, maximizing revenue calls for revealing low bids while at the same time pooling higher bids. Disclosure of low value implies that the auctioneer again prefers to disclose whether a bidder participated.

A natural follow-up question concerns the extent to which our results can be generalized to a richer class of mechanisms. Beyond mechanism design, that could provide new and exciting perspectives in applied fields such as advertising, marketing science and industrial organization. For instance, the literature on conspicuous consumption (e.g., Bagwell and Bernheim (1996) and Corneo and Jeanne (1997)) studies product markets where the consumption value depends on the belief of a social contact. A profit-maximizing seller will try to exploit this by tailoring its product line and prices to the information revealed by the consumer's choice. This potentially leads to new insights about consumer behavior and firm strategies that exploit signaling concerns.[21]

Declaration of competing interest

We declare that we have no relevant or material financial interests that relate to the research described in this paper.

Appendix A. Proof of Proposition 5

Consider the following variant of a first-price auction: Bidders submit non-negative bids, the bidder submitting the highest bid wins and with exogenous probability $1-\varepsilon$ makes no payment, but pays his own bid with probability ε.[22] Every bidder has to pay the entry fee φ before submitting a bid. The expected profit of a bidder of type v upon entering the auction and bidding as if he was type v' is

$$\Pi(v|v') = G(v')(v - \varepsilon\beta(v')) + \mathcal{W}_\tau(v') - \varphi.$$

From the first-order condition we get

$$\beta^\star(v) = \frac{1}{\varepsilon}\beta^{\mathcal{M}}(v) + \frac{\mathcal{W}_\tau(v) - \mathcal{W}_\tau(\tau)}{\varepsilon G(v)},$$

where $\beta^{\mathcal{M}}$ is the bidding strategy in a first-price auction without signaling and entry fee that induces only types above τ to participate. Note that we have used the fact that $\beta(\tau) = 0$, which is true because in equilibrium type τ only wins the auction when no other bidder enters and is thus not willing to bid a strictly positive amount. Furthermore, to induce participation for all types above τ the fee has to satisfy

$$\mathcal{W}_{\tau,0} = G(\tau)\tau + \mathcal{W}_\tau(\tau) - \varphi \quad \Leftrightarrow \quad \varphi = G(\tau)\tau + \mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}.$$

The revenue is thus given by

$$\begin{aligned} Rev^\varepsilon(\tau) &= n(1-F(\tau))\varphi + (1-F^n(\tau))\mathbb{E}\left[\varepsilon\beta^\star(V_1)|V_1 \geq \tau\right] \\ &= n(1-F(\tau))\big(G(\tau)\tau + \mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}\big) + \int_\tau^{\overline{v}} \left(\beta^{\mathcal{M}}(s) + \frac{\mathcal{W}_\tau(s) - \mathcal{W}_\tau(\tau)}{G(s)}\right)\mathrm{d}F^n(s) \\ &= Rev^{\mathcal{M}}(\tau) + n(1-F(\tau))(\mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}) + n\int_\tau^{\overline{v}} \big(\mathcal{W}_\tau(s) - \mathcal{W}_\tau(\tau)\big) f(s)\mathrm{d}s \\ &= Rev^{\mathcal{M}}(\tau) + n\left[F(\tau)\mathcal{W}_{\tau,\emptyset} + \int_\tau^{\overline{v}} \mathcal{W}_\tau(s)f(s)\mathrm{d}s - \mathcal{W}_{\tau,\emptyset}\right]. \end{aligned} \tag{12}$$

Note that

[20] Rayo and Segal (2010) study such a sender–receiver game. The receiver chooses a binary action $a \in \{0,1\}$. Choosing $a = 0$ yields a fixed utility r which is distributed according to some distribution function G. Choosing $a = 1$ yields utility θ, where θ is the sender's private information. The sender wants to maximize the probability of choosing $a = 1$. Hence, the sender's reduced form utility is $G(\mathbb{E}(\theta))$.

[21] See Rayo and Segal (2010) and Friedrichsen (2018) for analyses into that direction.

[22] The superscript A is omitted in the proof, as it goes through a specific first-price auction.

$$\mathcal{W}_\tau(s) = \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{ F_k(s|\tau)\cdot\Big(\varepsilon\Phi(s) + (1-\varepsilon)\Phi\big(v_{W,k}\big)\Big)$$

$$+ \big(1-F_k(s|\tau)\big)\cdot\Big(\varepsilon\int_s^{\overline{v}} \Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) + (1-\varepsilon)\Phi\big(v_{L,k}\big)\Big)\Big\}$$

$$= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{ F_k(s|\tau)\Phi\big(v_{W,k}\big) + \big(1-F_k(s|\tau)\big)\,\Phi\big(v_{L,k}\big)\Big\}$$

$$-\varepsilon\sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{ F_k(s|\tau)\Big(\Phi(s) - \Phi\big(v_{W,k}\big)\Big)$$

$$+(1-F_k(s|\tau))\Big(\int_s^{\overline{v}} \Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) - \Phi\big(v_{L,k}\big)\Big)\Big\},$$

where $\mathcal{B}_{n-1,F(\tau)}(k-1) := \binom{n-1}{k-1}F(\tau)^{n-k}(1-F(\tau))^{k-1}$, $v_{W,k} := \mathbb{E}[V_1|V_k \geq \tau > V_{k+1}]$, $v_{L,k} := \mathbb{E}[V|V_1 > V \geq V_k \geq \tau > V_{k+1}]$, $v_{L,k}(s) := \mathbb{E}[V|V_1 = s, s > V \geq V_k \geq \tau > V_{k+1}]$ and $F_k(v|\tau) := \big(\frac{F(v)-F(\tau)}{1-F(\tau)}\big)^{k-1}$ for all $k = 1,\dots,n$ denotes the conditional probability of the maximum of the $k-1$ other bids if all of these exceed τ. Hence,

$$\int_\tau^{\overline{v}} \mathcal{W}_\tau(s)f(s)\mathrm{d}s = \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{ \tfrac{1}{k}\Phi\big(v_{W,k}\big) + \tfrac{k-1}{k}\Phi\big(v_{L,k}\big)\Big\}$$

$$-\varepsilon\sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{ \int_\tau^{\overline{v}} F_k(s|\tau)\Big(\Phi(s) - \Phi\big(v_{W,k}\big)\Big)f(s)\mathrm{d}s$$

$$+\int_\tau^{\overline{v}} (1-F_k(s|\tau))\Big(\int_s^{\overline{v}} \Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) - \Phi\big(v_{L,k}\big)\Big)f(s)\mathrm{d}s\Big\}$$

$$= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{ \tfrac{1}{k}\Phi\big(v_{W,k}\big) + \tfrac{k-1}{k}\Phi\big(v_{L,k}\big)\Big\} - \varepsilon C,$$

where by concavity of Φ and compactness of the support of the bidders' valuations we have $C > 0$ and finite. Plugging the above expression back into (12) and noting that $\mathcal{W}_{\tau,\emptyset} = \mathbb{E}[V|V < \tau]$ (because participation is observable) yields $Rev^\varepsilon(\tau) = Rev^P(\tau) - \varepsilon C \to Rev^P(\tau)$ as $\varepsilon \to 0$.

Data availability

No data was used for the research described in the article.

References

Bagwell, L.S., Bernheim, B.D., 1996. Veblen effects in a theory of conspicuous consumption. Am. Econ. Rev. 86 (3), 349–373.
Bénabou, R., Tirole, J., 2006. Incentives and prosocial behavior. Am. Econ. Rev. 96 (5), 1652–1678.
Bos, O., Pollrich, M., 2023. Auctions with signaling bidders: Optimal design and information disclosure. ZEW working paper.
Bos, O., Truyts, T., 2021. Auctions with signaling concerns. J. Econ. Manag. Strategy 30, 420–448.
Calzolari, G., Pavan, A., 2006a. Monopoly with resale. Rand J. Econ. 37 (2), 362–375.
Calzolari, G., Pavan, A., 2006b. On the optimality of privacy in sequential contracting. J. Econ. Theory 130, 168–204.
Corneo, G., Jeanne, O., 1997. Conspicuous consumption, snobbism and conformism. J. Public Econ. 66 (1), 55–71.
Das Varma, G., 2003. Bidding for a process innovation under alternative modes of competition. Int. J. Ind. Organ. 21 (1), 15–37.
Dworczak, P., 2020. Mechanism design with aftermarkets: cutoff mechanisms. Econometrica 88 (6), 2629–2661.
Dworczak, P., Martini, G., 2019. The simple economics of optimal persuasion. J. Polit. Econ. 127 (5), 1993–2048.
Friedrichsen, J., 2018. Signals sell: Product lines when consumers differ both in taste for quality and image concern. Technical report, CRC TRR 190, Rationality and Competition.
Giovannoni, F., Makris, M., 2014. Reputational bidding. Int. Econ. Rev. 55 (3), 693–710.
Goeree, J.K., 2003. Bidding for the future: signaling in auctions with an aftermarket. J. Econ. Theory 108 (2), 345–364.
Holmström, B., 1999. Managerial incentive problems: a dynamic perspective. Rev. Econ. Stud. 66 (1), 169–182.
Jehiel, P., Moldovanu, B., 2000. Auctions with downstream interaction among buyers. Rand J. Econ. 31 (4), 768–791.
Jehiel, P., Moldovanu, B., 2001. Efficient design with interdependent valuations. Econometrica 69 (5), 1237–1259.
Katzman, B.E., Rhodes-Kropf, M., 2008. The consequences of information revealed in auctions. Appl. Econ. Res. Bull. 2, 53–87.
Krishna, V., 2009. Auction Theory, second edn. Academic Press, Elsevier.
Lewis, T.R., Sappington, D.E., 1989. Countervailing incentives in agency problems. J. Econ. Theory 49, 294–313.
Liu, T., 2012. Takeover bidding with signaling incentives. Rev. Financ. Stud. 25 (2), 522–556.
Mandel, B.R., 2009. Art as an investment and conspicuous consumption good. Am. Econ. Rev. 99 (4), 1653–1663.

Milgrom, P., Segal, I., 2002. Envelope theorems for arbitrary choice sets. Econometrica 70 (2), 583–601.
Milgrom, P., Weber, R., 1982. A theory of auctions and competitive bidding. Econometrica 50 (5), 1089–1122.
Molnar, J., Virag, G., 2008. Revenue maximizing auctions with market interaction and signaling. Econ. Lett. 99 (2), 360–363.
Myerson, R., 1981. Optimal auction design. Math. Oper. Res. 6 (1), 58–73.
Rayo, L., Segal, I., 2010. Optimal information disclosure. J. Polit. Econ. 118 (5), 949–987.
Riley, J.G., Samuelson, W.F., 1981. Optimal auctions. Am. Econ. Rev. 71 (3), 381–392.
von Scarpatetti, B., Wasser, C., 2010. Signaling in auctions among competitors. Mimeo.
Wan, Z., Beil, D.R., 2009. RFQ auctions with supplier qualification screening. Oper. Res. 57 (4), 934–949.
Wan, Z., Beil, D.R., Katok, E., 2012. When does it pay to delay supplier qualification? Theory and experiments. Manag. Sci. 58 (11), 2057–2075.