Auctions with Signaling Bidders: Optimal Design and Information Disclosure*

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Abstract

We study optimal auctions in a symmetric private values setting, where bidders have signaling concerns: they care about winning the object and a receivers inference about their type. Signaling concerns arise in various economic situations such as takeover bidding, charity auctions, procurement and art auctions. We show that auction revenue can be decomposed into the standard revenue from the respective auction without signaling concern, and a signaling component. The latter is the bidders' ex-ante expected signaling value net of an endogenous outside option: the signaling value for the lowest type. The revenue decomposition restores revenue equivalence between different auction designs, provided that the same information about bids is revealed. Revealing information about submitted bids affects revenue via the endogenous outside option. In general, revenue is not monotone in information revelation: revealing more information about submitted bids may reduce revenue. We show that any bid disclosure rule allowing to distinguish whether a bidder submitted a bid or abstained from participation minimizes the outside option, and therefore maximizes revenue.

Keywords: optimal auctions, revenue equivalence, Bayesian persuasion, information design

JEL classification: D44; D82

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1 Introduction

Since 1945, the *Hospices de Beaune*[1], in Burgundy (France), organizes an annual wine auction to raise money for local retirement houses and hospitals. In a special segment—the “pièce des Présidents”—a few barrels of wine are auctioned with the help of celebrities. Naturally, this segment draws considerable attention by the media, with extended press coverage. In the 2017 “pièce des Présidents” auction two barrels of *Corton Clos du Roi Grand Cru* were sold at a total price of €410,000. During the regular auction, the same wine realized prices ranging from €30,000 to €40,000 per barrel. Roughly speaking, public attention increased the price per barrel by 500%.[2] This wine auction is but one example of an auction where bidders have signaling concerns, i.e., bidders care about the object at sale but also about how they are perceived by others. For example takeover bidding is affected by the bidding firms' managers career concerns. The managers' future compensation is (partly) influenced by the inference potential employers draw from their performance in the takeover auction (Giovannoni and Makris, 2014). Also, competing firms pay the merger via issuing equity, and the value of equity depends on the market's belief about the value of the merged entity. The latter is the bidders' private information and imperfectly revealed during the auction (Liu, 2012). Signaling concerns are also present in procurement auctions with bidder qualification (Wan and Beil, 2009, Wan et al., 2012). The bidders' performance in past tenders serves as a signal of their intrinsic quality, affecting their chance for being qualified for future tenders. Bidding in license or spectrum auctions is carefully observed by the stock market. The auction outcome is highly relevant for later competition in the market and in itself for the financial well-being of the bidding companies.[3] Finally, Mandel (2009) identifies signaling as an important aspect for buying and investing in artwork.

In this manuscript, we study auction and information design when bidders care about the inference outsiders draw about their type. Each bidder values winning the object. In addition, each bidder has a signaling concern where she cares about the posterior belief about her type (i.e., her valuation for the object at sale). In our setting the seller runs an auction and publicly announces the winner. The auctioneer has two design tools to

[1] https://www.beaune-tourism.com/discover/hospices-de-beaune-wine-auction

[2] Similar patterns arose in the previous years. Data for 2016 and 2017 are available at http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/3869/14085/version/1/file/catalogue_resultats_2016.pdf and http://hospices-de-beaune.com/index.php?/hospicesdebeaune/content/download/4248/15476/version/1/file/Vente+des+vins+-+Catalogue+des+r%C3%A9sultats+2017.pdf

[3] The 5G spectrum auction held in Germany in 2019 was one of the most competitive auctions in European countries. It raised €6.5 billions. Drillisch Netz AG became the new fourth mobile operator with two bids of €1.07 billion together for 70 MHz of spectrum. In the meantime its share price increased by 11%. See https://www.reuters.com/article/us-germany-telecoms/germany-raises-6-55-billion-euros-in-epic-5g-spectrum-auction-idINKCN1TD27D?edition-redirect=in and https://www.bundesnetzagentur.de/SharedDocs/Pressemitteilungen/EN/2019/20190612_spectrumauctionends.html

capitalize on the bidders' willingness to pay for the object and their signaling concern. First, as in standard auction design, the payment rule specifying each bidder's payment as a function of the submitted bids, e.g., first or second price auction. The second design tool is a disclosure rule, revealing information about the submitted bids.[4] Such disclosure can range from no disclosure, where no further information is revealed, to fully disclosing all submitted bids alongside the bidders' identities.

We show a version of the revenue equivalence principle, stating that for a fixed disclosure rule the auctioneer's revenue is the same across all payment rules, i.e., auction designs. This result holds due to additive separability of the bidders' utility, which is the sum of her valuation for the object and the utility derived from signaling. However, the auctioneer's revenue does depend on the employed disclosure rule. Bidders have an endogenous outside option, arising from the (expected) signaling value a bidder realizes by not submitting a bid. Disclosure rules differ in the value of this endogenous outside option, and therefore in revenue. In particular, disclosing *more information* about bids can *decrease* revenue. To maximize revenue, the auctioneer strives at reducing the value of the outside option to its bare minimum. This corresponds to a disclosure rule that allows outsiders to distinguish between bidders that submitted a bid but did not win the object, from bidders that did not even submit a bid. As an example for such a disclosure rule, the auctioneer may run a full transparent auction in which all bidding is public. Also an English auction, in which the identities of active bidders are public reveals the optimal amount of information.

Information design is implicit in many real-world auctions, where laws govern the disclosure of information about submitted bids. Regulations for takeover bidding require public bids, that is every submitted bid has to be revealed to the public.[5] In contrast, in private procurement all details of the bidding process are treated as a trade secret. Merely the identity of the winning bidder becomes public, no information on bids and payments is revealed to the public. These examples capture the two extremes of information disclosure: full disclosure (takeover bidding) and no disclosure (private procurement). But information disclosure can also take forms inbetween these extremes. In public procurement regulations call for concealing individual bids, but the final price has to be published.[6] Because the final price is itself a function of submitted bids, there is some implicit information disclosure which depends on the selected payment rule. In a first-

[4]Formally, the auctioneer designs a *disclosure rule*, which maps a vector bids into publicly signals. See the model section for a formal definition and further examples.

[5]See Article 6 of Directive 2004/25/EC of the European Parliament and of the Council of 21 April 2004 on takeover bids: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32004L0025&from=EN (last accessed March 3rd 2021).

[6]See Articles 21 and 22 as well as Annex V Part D of Directive 2014/24/EU of the European Parliament and of the Council of 26 February 2014 on public procurement: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:02014L0024-20200101&from=EN (last accessed June 1st 2021).

price auction with public revelation of the winner's payment, outsiders infer the winner's bid from her payment. At the same time there is only noisy inference on losers' bids. Similarly, in a second-price auction with revelation of the winner's payment, outsiders observe the highest losing bid. This renders inference on *all* bidders noisy, because the identity of the highest losing bidder remains unknown. An all-pay auction provides an example of an auction format where information revelation is implicit (via prices) but still corresponds to full information disclosure as in the case of takeover bidding mentioned earlier.

In their seminal contributions Myerson (1981) and Riley and Samuelson (1981) show that—absent signaling concerns—every standard auction yields the same revenue. This is not necessarily the case when bidders care for signaling. Giovannoni and Makris (2014), Bos and Truyts (2021) study environments where different auction formats and bid disclosure policies yield different auction revenue, while for instance Goeree (2003), Molnar and Virag (2008), Katzman and Rhodes-Kropf (2008) find revenue equivalence in their respective settings. In this manuscript we study the simultaneous design of the auction (i.e., the payment rule) and information disclosure. We ask which information disclosure is optimal, and how the two design tools interact, i.e., for which disclosure rules does revenue equivalence obtain.

Auctions with signaling concerns have been recently investigated by Giovannoni and Makris (2014) and Bos and Truyts (2021).[7] While these papers focus on specific auction formats and disclosure rules, our analysis covers all standard auctions and arbitrary bid-disclosure policies. Giovannoni and Makris consider first- and second price auctions that reveal the winner's identity together with four disclosure policies: no further information, disclosure of the highest bid, disclosure of the the second highest bid, and disclosure of all bids. As confirmed by our analysis, they show that the revenue depends only on the disclosure rule, but not on the auction format. Their analysis further shows that whether more ore less information shall be disclosed depends on the number of bidders and whether bidders signaling concern is increasing or decreasing. Our analysis challenges these findings by showing that the number of bidders plays no role, but how disclosure affects the signaling value from not participating. In an extension we add to this by showing that also the shape of the signaling functions matters, but not its slope.[8] Bos and Truyts compare second-price and English auctions when bidders have *linear* signaling concern. They assume that only the winner's identity and her own payment get revealed. In the light of our results, the difference in auction revenue stems from the difference in information revealed about a bidder's participation.

[7]There are also contributions about information transmission comparing specific auction formats followed by oligopoly competition. See, e.g., Goeree (2003), Das Varma (2003), Katzman and Rhodes-Kropf (2008) and von Scarpatetti and Wasser (2010).

[8]More precisely, the slope only matters insofar that steeply decreasing signaling functions impede existence of equilibria. This has been pointed out already by Giovannoni and Makris (2014).

Our paper is also related to the literature on mechanism design with aftermarkets. Calzolari and Pavan (2006*a*,*b*) study contracting environments where the agent participates in an aftermarket. They find conditions under which no information release to the aftermarket is optimal. Dworczak (2020) analyzes an auction environment with a very general aftermarket. He restricts the analysis to cut-off mechanisms in which the information revealed about the winner only depends on the losers' bids. These mechanisms rule out disclosure of information contained only in the winner's bid, such as the price in a first-price auction or the entire vector of payments in the all-pay auction, which we show is optimal in some cases. Also Molnar and Virag (2008) study auctions with an aftermarket, where only the winner has a signaling concern (and, consequently, there is no aftermarket if there is no winner). They show that it is optimal to reveal (conceal) the winner's type when the signaling incentive is convex (concave). In our setting all bidders care about how they are perceived, irrespective of whether they win. This brings about a novel decomposition of revenue into the standard non-signaling component and a signaling component, which can be analyzed using methods from information design. In addition, a new trade off arises from the bidders' endogenous outside option, implicitly given by the signaling value from abstention, which yields new insights in the concave and convex cases.

Information disclosure in auctions has first been analyzed in the setting of affiliated values by Milgrom and Weber (1982). Mechanism design problems with allocative and informational externalities have also been studied by Jehiel and Moldovanu (2000, 2001). The underlying assumption in this strand of literature is that an agent's valuation depends also on other agents' private information (and allocation). In our setting a bidder's utility is affected by the aftermarket's belief about her own valuation, while such beliefs have no impact in the literature on mechanism design with interdependent valuations.

The paper is organized as follows. Section 2 introduces the formal setting. In Section 3 we provide an illustrative example that outlines our main findings, and the non-monotone impact of disclosing additional information. Section 4 provides our main result, a detailed analysis for the case of linear signaling concerns. We restore revenue equivalence and derive optimal auctions. Section 5 studies extensions of our main model to cover general mechanisms and non-linear signaling concerns. We derive optimal auctions when the signaling concerns are convex, or concave, and uncover a novel trade-off in the latter case. Section 6 concludes.

2 Formal Setting

We consider n bidders, who bid for a single object in an auction, and also care about the inference of an outside observer about their type.

Bidder i's valuation for the object (her 'type'), is denoted V_i, and is assumed i.i.d. and drawn according to a distribution function F with support on $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$. Let $f \equiv F'$ denote the density function, $G \equiv F^{n-1}$ the distribution function of the highest order statistic among $n-1$ remaining valuations and $g \equiv G'$ the corresponding density function. Bidder i's realization of V_i, denoted v_i, is her private information, but the number of bidders and the distribution F are common knowledge. The auctioneer's value for the object is zero.

We consider *standard auctions* with reserve in which each bidder submits a (non-negative) bid b_i, the bidder who places the highest bid above the reserve $r \geq 0$ wins (ties broken at random), and bidder i's payment p_i depends on the entire vector of bids, i.e., $p_i(b_1, \ldots, b_n)$. We allow bidders to abstain from participation, which we formally model as 'bidding' $b_i = \emptyset$. Hence, the bid space is $B \coloneqq [r, \infty) \cup \{\emptyset\}$. A bidder who abstains does not make a payment and never wins the object (also if all other bidders abstain as well).

After the auction was conducted, the winning bidder's identity is publicly revealed. Revelation of the winner's identity is not a design tool in most auction environments, e.g., in takeover or procurement auctions.[9] In addition, the auctioneer chooses an *information disclosure policy* revealing information about the submitted bids. Formally, an information disclosure policy (σ, S) consists of a signal space S and a mapping $\sigma : B^n \to \Delta S$. Upon submitting bids $b_1, \ldots, b_n$ a signal $s \in S$ is drawn (potentially at random) from the distribution $\sigma(b_1, \ldots, b_n) \in \Delta S$. This formulation encompasses as extremes (i) a full revelation policy, where $S = B^n$ and $\sigma(b_1, \ldots, b_n) = \mathbb{I}_{(b_1,\ldots,b_n)}$, and (ii) a no revelation policy, where, e.g., $S = \{s\}$ is a singleton.[10] Many real-world examples of auctions do not feature explicit bid revelation, but implicit revelation via observable prices. For example, guidelines on public procurement demand that the final price is public.[11] Our model captures this as follows: for any standard auction let $S = \mathbb{R}^n_+$, and $\sigma(b_1, \ldots, b_n) = \mathbb{I}_{\left\{(p_1(b_1,\ldots,b_n),\ldots,p_n(b_1,\ldots,b_n))\right\}}$. That is, the auctioneer reveals each bidders' payment, and thus implicitly some information about submitted bids.[12] We denote $\mathcal{O} \coloneqq (i^\star, s)$ the public outcome of the auction, where $i^\star$ is the winner's identity and s the signal revealed by the auctioneer.

Each bidder cares about winning the object, and about the inference of an outside

[9] Only in some art auctions it is commonplace to conceal the winner's identity.

[10] Technically there are many alternative variants for specifying these policies. E.g., no revelation can be obtained via an arbitrary signal space and a degenerate distribution σ which reveals the same signal, irrespective of the actual bids.

[11] See also Footnote 6.

[12] For instance, in a first-price auction the revelation of the winner's payment essentially reveals the winner's bid. In a second-price auction, the revelation of the winner's payment reveals that (i) the winner's bid was weakly higher, (ii) some losing bidder placed exactly this bid, (iii) all losing bidders placed a weakly lower bid. In an all-pay auction revelation of all payments is equivalent to revelation of all bids.

observer, the 'receiver', about her type. This receiver can represent, e.g., the general public or press, business contacts or acquaintances of the bidder, or experts related to the object at sale. The receiver observes the auction outcome $\mathcal{O}$, but no further information about the bidding process, and forms a posterior belief about each bidder's type, denoted as $\mu_i(\mathcal{O})$. We assume that a bidder's utility depends *linearly* on the posterior mean of the receiver's belief about the bidder's type.[13] A bidder's utility is given by

$$u_i(v_i, \mathcal{O}) = \begin{cases} v_i - p_i + \lambda \mathbb{E}\big(V_i|\mathcal{O}\big), & \text{if } i = i^\star, \\ -p_i + \lambda \mathbb{E}\big(V_i|\mathcal{O}\big), & \text{if } i \neq i^\star. \end{cases} \tag{1}$$

The parameter $\lambda \geq 0$ measures the strength of the bidders' signaling concern. This signaling function represents a reduced form of a (continuation) game in which the receiver chooses an action that directly affects the bidder's payoff. Note that a bidder's utility is not affected by the receiver's belief about other bidders' types. For instance, from an individual bidder's perspective it is equivalent to have either a different or the same receiver for each bidder.

Any standard auction with information disclosure defines a signaling game among bidders and the receiver. We consider symmetric perfect Bayesian equilibrium, consisting of the bidders' bidding strategies $\beta : [\underline{v}, \overline{v}] \to B$ and the receiver's belief $(\mu_1, \ldots, \mu_n)$. Each bidder's bidding strategy is optimal, given the other bidders' bidding and the receiver's beliefs. Also, the receiver's beliefs are Bayesian consistent with the bidding strategy. In our analysis we focus on equilibria in which (participating) bidders use strictly increasing bidding functions. Restricting to symmetric equilibria follows the usual practice in the literature, given the symmetric environment that we study. Only symmetric and strictly increasing equilibria implement an efficient allocation of the object to the bidder with the highest valuation. Besides this effiency-driven motivation, it is possible to show that if the bidders' signaling concern is not too strong (compared the payoff from winning) all equilibria of a standard auction have to be in increasing strategies. It thus allows for a straightforward comparison with the no-signaling benchmark.

We conclude this section with two applications of our formal setting.

Takeover bidding with career concerns. Suppose the bidders are the managers of firms, bidding in a takeover. The profit of the merged firm, if manager i's firm wins, is v_i. We assume that v_i is a proxy for the manager's ability, i.e., it is the manager who makes the merger profitable, and the efficient takeover involves the most talented manager. Managers maximize their firm's profit, i.e., conditional on winning the difference between post-merger profit v_i and takeover price p. This is reasonable, for instance if the

[13] Note that we do not assume that a bidder's type v_i *directly* affects the receiver's payoff. The receiver may care about some other characteristic of the bidder, which is correlated with the bidder's type. See also the examples at the end of this section.

manager's compensation bases on the firm's profit. After the takeover auction concluded, a competitive labor market opens for the managers' services. As in the models of career concerns (e.g, Holmström (1999)), the wage to manager i in this competitive market with risk-neutral firms equals the manager's expected ability, i.e., we have $w_i^\star = \mathbb{E}(V_i|\mathcal{O})$.
Image concerns in a charity auction. Suppose a good with common value $\Gamma > 0$ is sold in a charity auction. Bidders derive value from obtaining the good and from contributing to the charity. Paying p dollars to the charity yields utility νp, where $\nu < 1$. Hence, obtaining the good while paying a price p yields $\Gamma - p + \nu p$, while only paying p without obtaining the good yields $-p+\nu p$. Transforming appropriately yields the bidder's utility is $v\Gamma - p$, and $-p$ respectively (i.e., multiplying through by $v = 1/(1-\nu)$). In addition to the described utility from direct participation in the charity, bidders have preferences for their social image. Following Bénabou and Tirole (2006), each bidder's utility also depends additively on the expected altruism, i.e., on societies expected value of v (resp., ν). Combining the bidder's direct utility from participating in the auction with the utility from her social image yields the utility as given by Equation (1).

3 Illustrative example

Before we begin with our analysis, we provide an example that highlights some of our main findings. Consider a first-price auction with two bidders, whose valuations are iid drawn from the uniform distribution on $[0, 1]$. The auctioneer sets a reserve price r_τ to induce participation threshold $\tau \in [0, 1)$, i.e., a bidder participates in the auction if and only if her valuation exceeds τ.

It is familiar to analyze the first-price auction absent any signaling concern (e.g., $\lambda = 0$). The auctioneer sets $r_\tau = \tau$ and the bidders use the bidding function

$$\beta_\tau^{\mathcal{M}}(v) = \begin{cases} \frac{v}{2} + \frac{\tau^2}{2v}, & v \geq \tau, \\ 0, & v < \tau. \end{cases} \tag{2}$$

The auction results in a revenue $Rev^{\mathcal{M}}(\tau) = \frac{1}{3} - \frac{\tau^3}{3} + \tau^2(1-\tau)$.

3.1 All bids disclosed

From now on bidders have signaling concerns, i.e., we have $\lambda > 0$. First consider the case where all bids are disclosed. With participation threshold τ bidders who did not place a bid are perceived as average type $\tau/2$. Under strictly increasing bidding strategies, all participating bidders' types can be inferred from their bids. A bidder of type τ just bids the reserve price r_τ^A, because she only wins if there is no other active bidder, hence

$\tau(\tau - r^A_\tau) + \lambda\tau = \lambda\frac{\tau}{2}$, i.e.,

$$r^A_\tau = \tau + \frac{\lambda}{2}.$$

A bidder of type $v \geq \tau$ upon placing bid $b = \beta(z)$ for some $z \geq \tau$ obtains expected payoff

$$z\big(v - \beta(z) + \lambda z\big) + (1-z)\lambda z = z\big(v - \beta(z)\big) + \lambda z.$$

Using standard arguments, the equilibrium bidding strategy can be shown to be

$$\beta^A_\tau(v) = \begin{cases} \beta^{\mathcal{M}}(v) + \frac{\lambda}{2} + \lambda\frac{v-\tau}{2v}, & v \geq \tau, \\ \emptyset, & v < \tau. \end{cases}$$

There is over-bidding as compared to the auction without signaling concern for any $\lambda > 0$, raising the seller's revenue to

$$Rev^A(\tau) = Rev^{\mathcal{M}}(\tau) + \lambda(1-\tau)$$

Notice that the auctioneer's revenue from signaling strictly decreases with τ. This is due to the fact that the auctioneer can only extract signaling rents from bidders who actually participate in the auction.

3.2 Only winner's bid disclosed

Next consider the case where only the winner's bid is disclosed, alongside the winner's identity.As before bidders follow a strictly increasing bidding strategy, and participate whenever their type lies above τ. If there is no winner (and thus no winning bid) both bidders are perceived as average type $\tau/2$. In case there is a winner, the winner is perceived as her true type, which can be inferred from her bid because of the increasing bidding strategy. Denote $\tilde{v}$ the winner's revealed type. The loser is then perceived as average type $\tilde{v}/2$.[14] The reserve price r^W_τ inducing participation threshold τ is[15]

$$r^W_\tau = \tau + \lambda\frac{\tau}{2},$$

and the bidding strategy is

$$\beta^W_\tau(v) = \begin{cases} \beta^{\mathcal{M}}_\tau(v) + \frac{\lambda}{4v}\left(3v^2 - \tau^2\right), & v \geq \tau, \\ \emptyset, & v < \tau. \end{cases}$$

[14] Hence, not participating in the auction yields expected payoff $\tau\lambda\frac{\tau}{2} + \int_\tau^1 \lambda\frac{\tilde{v}}{2}\mathrm{d}F(\tilde{v}) = \frac{\lambda}{4}\big(1+\tau^2\big)$.

[15] The reserve price is derived from type τ's indifference condition $\frac{\lambda}{4}\big(1+\tau^2\big) = \tau(\tau - r^W_\tau + \lambda\tau) + \int_\tau^1 \lambda\frac{\tilde{v}}{2}\mathrm{d}F(\tilde{v})$.

As before, there is over-bidding for every participating type compared to the first-price auction without signaling. However, for every $\tau < 1$ and every v we find that $\beta_\tau^W(v) < \beta_\tau^A(v)$. Finally, the revenue in this equilibrium is

$$Rev^W(\tau) = Rev^{\mathcal{M}}(\tau) + \lambda(1-\tau)\frac{1+\tau}{2}.$$

Given that bids are lower it is not surprising to see that also revenue is lower.

3.3 No bids disclosed

As a third variation consider the case where no bids are disclosed. Yet, the winner's identity is disclosed. If there is no winner both bidders are perceived as average type $\tau/2$. If there is a winner, the outside observer notices that the winner's type has to exceed τ. As a short-hand notation denote $\mathbb{W}$ the expected type of the winner, and $\mathbb{L}$ the expected type of the loser. Not participating yields the payoff $\tau \cdot \lambda\frac{\tau}{2} + (1-\tau)\cdot\lambda\mathbb{L}$. The reserve price r_τ^N that guarantees participation threshold τ is

$$r_\tau^N = \tau + \lambda\mathbb{W} - \lambda\frac{\tau}{2},$$

and the equilibrium bidding strategy is

$$\beta_\tau^N(v) = \begin{cases} \beta_\tau^{\mathcal{M}}(v) + \lambda\left(\mathbb{W} - \mathbb{L} + \frac{\tau}{v}\left(\mathbb{L} - \frac{\tau}{2}\right)\right), & v \geq \tau, \\ \emptyset, & v < \tau. \end{cases}$$

Again there is over-bidding as compared to the first-price auction without signaling concern. After some algebraic manipulations,[16] the revenue in this equilibrium is

$$Rev^N(\tau) = Rev^{\mathcal{M}}(\tau) + \lambda(1-\tau)\frac{1+6\tau-\tau^2}{3(1+\tau)}.$$

Comparing to the revenue when all bids are disclosed it is straightforward to verify that $Rev^N(\tau) < Rev^A(\tau)$ for all $\tau \in [0,1)$.

3.4 Revenue Comparison and Optimal Disclosure

As discussed before, we have that the auction that discloses all submitted bids yields higher revenue than both the auction that discloses only the winner's bid, and the auction that discloses no bids. Comparing the revenues of the latter two auctions is less

[16] It uses the law of iterated expectation $\mathbb{E}(V) = \tau^2 \cdot \frac{\tau}{2} + (1-\tau^2)\left(\frac{1}{2}\mathbb{W} + \frac{1}{2}\mathbb{L}\right)$.

straightforward. We have that

$$Rev^W(\tau) > Rev^N(\tau) \qquad \Leftrightarrow \qquad \tau < \frac{1}{5}$$

In words, starting from no bid disclosure, disclosing the winner's bid in addition only improves revenue when the participation threshold is sufficiently low. However, disclosing all bids always yields higher revenue. The auction revenue is not (everywhere) increasing in the amount of information disclosed by the auctioneer. Figure 1 exhibits the three revenue curves for the cases $\lambda = 0.4$ and $\lambda = 0.8$. Apart from the ranking of revenues, it reveals another remarkable feature. The disclosure policy has a direct impact on the *optimal* participation threshold. Upon disclosing all bids full participation is optimal, while both alternative disclosure policies yield interior thresholds.

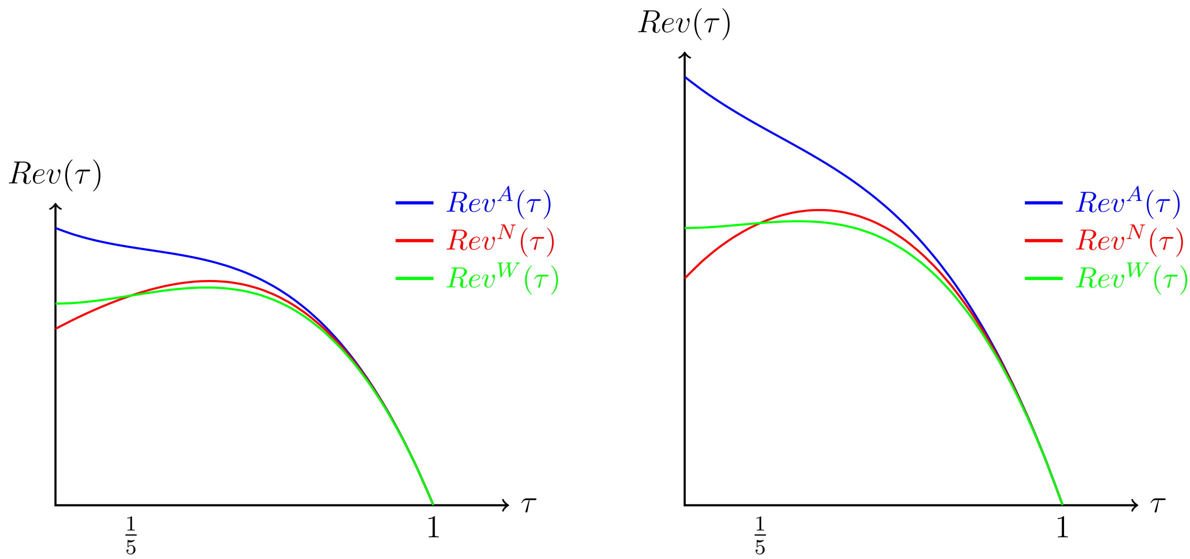

Figure 1: Revenue curves for the case $\lambda = 0.4$ and the three disclosure policies.

One of our main findings will be that the information about non-participating bidders is crucial for determining auction revenue. To see this in the light of our examples, consider again the case where the winner's bid is disclosed. Now assume that in addition the auctioneer reveals whether the loser (if there is one) placed a bid above the reserve price, but not the exact bid.

Under the described disclosure policy non-participant's expected utility reduces to $\lambda\frac{\tau}{2}$. The reserve price r_τ^{WP} is

$$r_\tau^{WP} = \tau - \frac{\lambda}{2} + \frac{\lambda(1+\tau)^2}{4\tau} = r_\tau^W + \frac{\lambda}{4\tau}(1-\tau^2),$$

where we can see that the additional disclosure allows for an increase of the reserve price. Naturally, this increase leads to an increase of all bids. Formally, the equilibrium bidding strategy is

$$\beta_\tau^{WP}(v) = \begin{cases} \beta_\tau^{\mathcal{M}}(v) + \frac{\lambda}{2v}\left(\frac{1}{2} + \frac{3}{2}v^2 - v\tau\right), & v \geq \tau, \\ 0, & v < \tau. \end{cases}$$

Computing the revenue from this equilibrium yields

$$Rev^{WP}(\tau) = Rev^{\mathcal{M}}(\tau) + \lambda(1-\tau) = Rev^{A}(\tau).$$

Hence, disclosing in addition whether a bidder participated in the auction (i.e., placed a bid above the reserve price) increases the auctioneers revenue. It yields the same revenue as disclosing all bids, which can be shown yields the maximal revenue across all disclosure policies (see Proposition 2 below). It is straightforward to verify that disclosing participation in the auction that discloses no bids also yields revenue $Rev^A(\tau)$. It is not important which information on bids gets disclosed, but which information on the bidders' participation decisions is disclosed. While the former has redistributive effects, i.e., how signaling value is distributed across types, the latter directly affects the auctioneers revenue.

4 Analysis

For the first part of the analysis we fix some standard auction cum disclosure policy A. We initially focus on equilibria where (i) a bidder participates if and only if her valuation exceeds τ, and (ii) (participating) bidders follow a strictly increasing bidding strategy. We later extend the argument to all possible equilibria of the respective auction.

Denote $Rev^{\mathcal{M}}(\tau)$ the revenue of auction A if bidders have no signaling concern, i.e.,

$$Rev^{\mathcal{M}}(\tau) = n\int_\tau^{\overline{v}}\left(G(\tau)\tau + \int_\tau^v g(x)x\mathrm{d}x\right)\mathrm{d}F(v).$$

Further let $\mathcal{W}_{\tau,\emptyset}^A$ denote the (interim) expected signaling value of a bidder who abstains from participation. We have that

Proposition 1 (Revenue decomposition)**.** *Consider a standard auction A, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. The revenue in this auction is given by*

$$Rev^{\mathcal{M}}(\tau) + n\left(\lambda\mathbb{E}(V) - \mathcal{W}_{\tau,\emptyset}^A\right). \tag{3}$$

Proof. We expand standard arguments from auction theory to our setting with signaling bidders (Riley and Samuelson, 1981, Krishna, 2009). Denote $m_\tau^A(v)$ the expected payment and $\mathcal{W}_\tau^A(v)$ the expected value from signaling of a bidder with valuation v. By assumption, we have $m^A_\tau(v) \equiv 0$ and $\mathcal{W}^A_\tau(v) \equiv \mathcal{W}^A_{\tau,\emptyset}$ for all $v < \tau$. Now consider a bidder with valuation $v \geq \tau$. His expected payoff from mimicking type $\widetilde{v} \geq \tau$ is

$$\Pi(v, \widetilde{v}) = G(\tilde{v})v - m^A(\tilde{v}) + \mathcal{W}^A_\tau(\tilde{v}). \tag{4}$$

At equilibrium the bidder's payoff is $\Pi(v) := \Pi(v, v)$ and using the envelope theorem[17] it follows that

$$G(v)v - m^A(v) + \mathcal{W}^A_\tau(v) = \Pi(v) = \Pi(\tau) + \int_\tau^v G(x)\, \mathrm{d}x. \tag{5}$$

A bidder with valuation τ is indifferent whether to participate, if

$$\Pi(\tau) = \mathcal{W}^A_{\tau,\emptyset}. \tag{6}$$

Using (5) and (6) we express the *interim* expected payment of a bidder as follows

$$m^A(v) = G(v)v - \int_\tau^v G(x)\, \mathrm{d}x + \mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset} = G(\tau)\tau + \int_\tau^v g(x)x\, \mathrm{d}x + \mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}, \tag{7}$$

where the second equality uses integration by parts. With this we can write the auctioneer's revenue as

$$\begin{aligned} Rev^A(\tau) &= n\int_{\underline{v}}^{\overline{v}} m^A(v)\mathrm{d}F(v) = n\int_\tau^{\overline{v}} m^A(v)\mathrm{d}F(v) \\ &= Rev^{\mathcal{M}}(\tau) + n\int_\tau^{\overline{v}} \left(\mathcal{W}^A_\tau(v) - \mathcal{W}^A_{\tau,\emptyset}\right)\mathrm{d}F(v) \\ &= Rev^{\mathcal{M}}(\tau) + n\left(F(\tau)\mathcal{W}^A_{\tau,\emptyset} + \int_\tau^{\overline{v}} \mathcal{W}^A_\tau(v)\mathrm{d}F(v) - \mathcal{W}^A_{\tau,\emptyset}\right) \\ &= Rev^{\mathcal{M}}(\tau) + n\left(\lambda\mathbb{E}(V) - \mathcal{W}^A_{\tau,\emptyset}\right). \end{aligned}$$

The last equality uses linearity of the bidders' signaling concern and the law of iterated expectation. □

The Proposition shows that revenue in the auction with signaling bidders can be decomposed into the revenue of the respective auction without signaling concern, and the revenue that arises solely from the bidders' signaling concern. Standard auction theory shows that the former is independent of the details of the auction, as long as it facilitates an equilibrium in strictly increasing bidding strategies.

It is immediate from Proposition 1 that differences in auction revenue are solely due to the signaling value to non-participating bidders $\mathcal{W}^A_{\tau,\emptyset}$. Varying disclosure policies lead to a mere redistribution of total signaling value across bidder types. The auctioneer extracts

[17] See Milgrom and Segal (2002). The objective in (4) is differentiable in v, and its derivative $G(\widetilde{v})$ is uniformly bounded.

signaling value only from participating bidders (irrespective of the re-distribution), where the signaling value from not participating serves as an endogenous outside option.

Using the disclosure policy (σ, S) to shift signaling value to types $v \geq \tau$ therefore increases revenue for two reasons: (i) it increases the signaling value of types from which this value can be extracted, and (ii) it lowers the bidders' outside option and allows for extracting *more* of their signaling value. Consequently, revealing whether a bidder participated maximizes the auctioneer's revenue.

Proposition 2 (Optimal disclosure)**.** *Consider a standard auction in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. For any disclosure policy, the associated revenue is bounded by*

$$Rev^{\mathcal{M}}(\tau) + n\lambda\big(\mathbb{E}(V) - \mathbb{E}(V|V < \tau)\big). \tag{8}$$

Moreover, the bound is attained for any disclosure policy that reveals whether a bidder participated.

Proof. We have $\mathcal{W}^{A}_{\tau,\emptyset} \geq \lambda\mathbb{E}(V|V < \tau)$, because by assumption all types below τ do not participate, and are hence lumped together. Plugging this into (3) yields the bound in (8). Upon disclosing whether a bidder participated it is straightforward that the latter inequality turns into an equality. Hence, the revenue bound is tight. □

There are several practical ways for implementing the optimal disclosure. First, disclosing all bids necessarily reveals whether an individual bidder actually placed a bid. For example, regulations in takeover bidding demand all bids to be public. Second, the auctioneer discloses a list of bidders that submitted a valid bid, i.e., a bid above the reserve price. Third, the auctioneer charges an *entry fee* and reveals each bidder's final payment. This way, the bidding process remains confidential, but participation decisions become public. In particular, the latter two variants of information disclosure do not necessarily reveal the entire ranking of bidders.

Example 1. We highlight the usefulness of Proposition 2 in the light of the examples provided in Section 3.

In a first-price auction with all bids published, individual participation decisions are public, and thus $\mathcal{W}^{A}_{\emptyset,\tau} = \mathbb{E}(V|V < \tau) = \tau/2$. Upon publishing the winner's bid and identity, we have that the signaling value of a non participating bidder is

$$\mathcal{W}^{W}_{\emptyset,\tau} = \tau \cdot \frac{\tau}{2} + \int_{\tau}^{1} \frac{\tilde{v}}{2} \mathrm{d}F(\tilde{v}) = \frac{1+\tau^2}{4} = \frac{\tau}{2} + \frac{(1-\tau)^2}{4}.$$

Upon only publishing the winner's identity, we have

$$\mathcal{W}^{N}_{\emptyset,\tau} = \tau \cdot \frac{\tau}{2} + (1-\tau)\mathbb{L}.$$

Comparison yields $\mathcal{W}^A_{\emptyset,\tau} < \mathcal{W}^W_{\emptyset,\tau}$ and $\mathcal{W}^A_{\emptyset,\tau} < \mathcal{W}^N_{\emptyset,\tau}$ for all $\tau < 1$. Furthermore, $\mathcal{W}^N_{\emptyset,\tau} > \mathcal{W}^W_{\emptyset,\tau}$ if and only if $\tau < 1/5$. This confirms the revenue ranking obtained in Section 3.

Next we want to compare optimal levels of participation among situations with and without signaling concerns. We focus on auctions with optimal information disclosure, i.e., auctions which make participation decisions public, as these attain the maximal revenue determined in Proposition 2. Denote with $\tau^\star(\lambda)$ the optimal level of participation under signaling strength λ.[18] Note that $\tau^\star(0) = \tau^{\mathcal{M}}$, where $\tau^{\mathcal{M}}$ denotes the optimal participation cut-off in an auction without signaling concerns. The following corollary shows that the optimal level of participation (weakly) increases in the signaling strength and there exists a finite threshold of signaling strength which makes full participation optimal.

Corollary 1 (Optimal Participation)**.** *Assume virtual valuations are increasing. We have that*

(*i*) $\tau^\star(\lambda) \le \tau^\star(\lambda') < \tau^{\mathcal{M}}$ *for all* $\lambda > \lambda' > 0$.

(*ii*) *There exists* $\overline{\lambda}$ *such that* $\tau^\star(\lambda) = \underline{v}$, *for all* $\lambda > \overline{\lambda}$.

Proof. From Riley and Samuelson (1981) we know that

$$\big(Rev^{\mathcal{M}}\big)'(\tau) = nF^{n-1}(\tau)\big(1 - F(\tau) - \tau f(\tau)\big) = -nf(\tau)F^{n-1}(\tau)\left(\tau - \frac{1-F(\tau)}{f(\tau)}\right).$$

Provided that virtual valuations $v - \frac{1-F(v)}{f(v)}$ are strictly monotone we have that $Rev^{\mathcal{M}}(\tau)$ has a unique maximum $\tau^{\mathcal{M}}$. Furthermore, $\big(Rev^{\mathcal{M}}\big)'(\tau) < 0$ for all $\tau > \tau^{\mathcal{M}}$, and $\big(Rev^{\mathcal{M}}\big)'(\tau) > 0$ for all $\tau \in (\underline{v}, \tau^{\mathcal{M}})$. Together with the observation that $\mathbb{E}(V|V < \tau)$ strictly increases in τ, (*i*) follows. To prove (*ii*) note that the derivative of $Rev^{\mathcal{M}}(\tau)$ is bounded. Hence, as soon as λ becomes sufficiently large we have that $\lambda \frac{\partial}{\partial \tau}\mathbb{E}(V|V < \tau) > \big(Rev^{\mathcal{M}}\big)'(\tau)$ for all $\tau > \underline{v}$ and thus full participation maximizes revenue in the auction with signaling concerns. □

5 Discussion

This section studies extensions of our model and discusses equilibrium selection, other mechanisms, and (non) robustness of the results about linear signaling concerns.

5.1 Equilibrium Selection

Our focus in the analysis so far has been on equilibria with strictly increasing bids. These equilibria are the natural focus in auction theory, because they yield the efficient allocation

[18] In general there may not be a unique optimal level of participation. Our assumption of increasing virtual valuations in Corollary 1 guarantees both existence and uniqueness of $\tau^\star(\lambda)$.

(net of not allocating the object when all valuations are low). In this section we extend the analysis to all equilibrium outcomes that may arise in an auction with information disclosure. First, using standard arguments we show that equilibrium bidding has to be weakly increasing.

Lemma 1. *Fix some auction with information disclosure. Every symmetric equilibrium is in weakly increasing strategies.*

Proof. Fix types $v < v'$ and suppose b (b') is an equilibrium bid of type v (v'). Denote $H(b)$ the winning probability, $m(b)$ the expected payment, and $\mathcal{W}(b)$ the expected signaling value from bidding b. Equilibrium conditions yield

$$vH(b) - m(b) - \mathcal{W}(b) \geq vH(b') - m(b') - \mathcal{W}(b'),$$

and

$$v'H(b') - m(b') - \mathcal{W}(b') \geq v'H(b) - m(b) - \mathcal{W}(b).$$

Combining the two inequalities, we get $(v' - v)\big(H(b') - H(b)\big) \geq 0$, thus $H(b') \geq H(b)$, and therefore $b' \geq b$. □

Following the Lemma, every equilibrium of an auction with information disclosure gives rise to an interval of types $[0, \tau)$ that do not participate. In addition, the implemented allocation is weakly monotone, perhaps with bunching of (intervals of) adjacent types. But then the revenue formula obtained in Proposition 1 changes only in its first component. Most importantly, it remains optimal to reveal individual participation decisions. While this answers the questions of optimal information disclosure, it leaves open the question of the optimal auction design. Using insights from Myerson (1981) bunching is never optimal if the distribution F satisfies a regularity condition. In that case any auction that gives rise to an equilibrium with strictly increasing bid functions together with an optimal information disclosure yields the maximal revenue for the auctioneer.

5.2 General Mechanisms

When bidders have no signaling concern the restriction to auctions is known to be without loss if Myerson's regularity condition applies. A similar reasoning holds in our setting. Due to additive separability of the bidders' utility, the revenue can be decomposed into a component capturing the revenue absent any signaling concern, and a component capturing additional rents extracted due to the bidders' signaling concern. Regarding the first component it is then straightforward that under Myerson's regularity assumption implementing an allocation rule that allocates the object to the bidder with the highest valuation, if that valuation exceeds a critical threshold, is optimal. Maximizing the signaling value given this allocation is then the same exercise as in main body of this manuscript. Hence, under the regularity assumption our focus is not restrictive.

An element that arises upon considering general mechanisms that of belief extortion. In auction, a bidder who submits a bid that never wins the object will never have to make a payment. This way, non-participants make no payments but potentially benefit in terms of their signaling value from being lumped together with (some) participants. In general, the seller benefits from charging a fee also to non-participants, and disclosing whether a bidder was willing to pay that fee. Together with the belief-threat that bidders who did not pay this fee are of the lowest possible type with certainty, this allows the seller to extract additional rents from the bidders. Though this is theoretically feasible, we belief it is of little relevance in practice. Institutional constraints, such as payments are only made by parties that undergo a transaction (i.e., winner pays auctions) prevent the belief extraction sketched above.

5.3 Non-linear Signaling Concerns

In this section, we briefly consider non-linear signaling concerns. As before we assume a bidder's utility depends on the posterior mean $\mathbb{E}(V_i|\mathcal{O})$, but consider increasing signaling functions $\Phi(\mathbb{E}(V_i|\mathcal{O}))$ that need not be linear. An auction with information disclosure induces in equilibrium a distribution over posterior means. We denote by H^A_τ this distribution, in an equilibrium where bidders with valuation above τ participate and use a strictly increasing bidding strategy. Using the same arguments as for proving Proposition 1 we get the following Lemma.

Lemma 2. *Consider a standard auction A, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. The revenue in this auction is given by*

$$Rev^{\mathcal{M}}(\tau) + n\left(\int_{\underline{v}}^{\overline{v}} \Phi(\tilde{v})\mathrm{d}H^A_\tau(\tilde{v}) - \mathcal{W}^A_{\tau,\emptyset}\right). \tag{9}$$

Proof. In the proof of Proposition 1 we have shown that

$$Rev^A(\tau) = Rev^{\mathcal{M}}(\tau) + n\left(F(\tau)\mathcal{W}^A_{\tau,\emptyset} + \int_\tau^{\overline{v}} \mathcal{W}^A_\tau(v)\mathrm{d}F(v) - \mathcal{W}^A_{\tau,\emptyset}\right)$$

The law of iterated expectation implies that

$$F(\tau)\mathcal{W}^A_{\tau,\emptyset} + \int_\tau^{\overline{v}} \mathcal{W}^A_\tau(v)\mathrm{d}F(v) = \int_{\underline{v}}^{\overline{v}} \Phi(\tilde{v})\mathrm{d}H^A_\tau(\tilde{v}),$$

from which the claim follows. □

With non-linear signaling concerns the total signal value $\int_{\underline{v}}^{\overline{v}} \Phi(\tilde{v}) \mathrm{d}H_\tau^A(\tilde{v})$ crucially depends on the shape of Φ. Upon considering only this term in the auctioneer's objective, her problem is that of finding the distribution over posterior means maximizing the signaling value. In doing so, the auctioneer is constrained by (i) having to reveal the winner's identity, and (ii) being able to reveal only information that she gathered from the bidders. Following insights from information design (Dworczak and Martini, 2019) a distribution over posterior means H can be implemented via some disclosure rule if and only if $H_\tau^{\max}$ is a mean-preserving spread of H and H is a mean-preserving spread of $H_\tau^{\min}$, where the distributions $H_\tau^{\max}$ and $H_\tau^{\min}$ correspond to maximal, respectively minimal, information disclosure. Formally, we have that

$$H_\tau^{\max}(v) = \begin{cases} 0, & \text{if } \underline{v} \leq v < \mathbb{E}[V|V \leq \tau], \\ F(\tau), & \text{if } \mathbb{E}[V|V \leq \tau] \leq v \leq \tau, \\ F(v), & \text{if } \tau < v \leq \overline{v}. \end{cases} \tag{10}$$

and $H_\tau^{\min}$ is the discrete distributions on three values $\{x_1, x_2, x_3\}$ where

$$x_1 = \mathbb{E}(V|V \leq \tau), \quad x_2 = \mathbb{E}(V^{1:n}|V^{1:n} \geq \tau), \quad x_3 = \mathbb{E}(V|V^{1:n} \geq \tau, V^{1:n} > V)$$

with $V^{1:n}$ the highest order statistic among n. The respective probabilities are

$$h_\tau^{\min}(x_1) = F(\tau)^n, \quad h_\tau^{\min}(x_2) = \big(1 - F(\tau)^n\big)/n, \quad h_\tau^{\min}(x_3) = \big(1 - F(\tau)^n\big)\frac{n-1}{n}.$$

Distribution $H_\tau^{\max}$ stems from disclosing the true value of participating bidders, only lumping together non-participants. Distribution $H_\tau^{\min}$ arises from disclosing only the winner's identity, if there is one.

But the disclosure policy not only affects the total signaling value, but also the endogenous outside option $\mathcal{W}_{\tau,\emptyset}$. In general, what turns out optimal regarding on of these components may not be optimal regarding the other. We illustrate this by considering two special cases of Φ for which the information design problem sketched above has a particularly simple solution.

Convex signaling concern. When Φ is convex, the signaling value is maximal upon disclosing all available information, i.e., inducing the distribution over posterior means $H_\tau^{\max}$. This is achieved by revealing all bids, which indirectly reveals bidders' types due to the strictly increasing bid function. Coincidentally, such an information disclosure policy also minimizes the outside option $\mathcal{W}_{\tau,\emptyset}$ as already discussed in the previous section. Hence, the optimal information disclosure amounts to disclosing all bids. Upon doing so, the specific payment rule does not further affect the auctioneer's revenue, because the first term in the revenue does not depend on it.

Proposition 3 (Optimal auction under convexity)**.** *Consider a standard auction, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy. With convex signaling concern, the revenue in this auction is at most*

$$Rev^{\mathcal{M}}(\tau) + n \left(\int_{\tau}^{\overline{v}} \Phi(v) dF(v) - (1 - F(\tau))\Phi(\mathbb{E}[V|V \leq \tau]) \right). \tag{11}$$

Any standard auction with full revelation of all bids exhibits the described equilibrium and attains the revenue bound.

Proof. For every distribution over means H that can be imlemented we know that it $H^{\max}_{\tau}$ has to be mean-preserving spread of it. Convexity of Φ then implies that,

$$\int_{\underline{v}}^{\overline{v}} \Phi(v) dH(v) \leq \int_{\underline{v}}^{\overline{v}} \Phi(v) dH^{\max}_{\tau}(v) = F(\tau)\Phi(\mathbb{E}[V|V \leq \tau]) + \int_{\tau}^{\overline{v}} \Phi(v) dF(v). \tag{12}$$

Together with our previous observation that $\mathcal{W}^{A}_{\tau,\emptyset} \geq \Phi(\mathbb{E}[V|V \leq \tau])$, with equality upon disclosing all bids, the revenue bound (11) follows.

To show the revenue bound (11) can be attained, consider a standard auction with a full disclosure policy, i.e., where all bids are disclosed. Denote $\beta^{\mathcal{M}}(v)$ the respective equilibrium bid in the auction without signaling concern, and $m^{\mathcal{M}}(v)$ the respective expected payment upon bidding as type v, i.e., placing the bid $\beta^{\mathcal{M}}(v)$. Now define $\beta(v) = \beta^{\mathcal{M}}(v) + x(v)$ such that for the expected payment in a separating equilibrium of the auction with signaling we have $m(v) = m^{\mathcal{M}}(v) + \Phi(v)$. By definition, β is strictly increasing. Hence, revealing the bid in fact reveals the type. It remains to show that bidding according to β indeed forms an equilibrium. Specify the system of beliefs such that (i) a bidder who did not participate (i.e., bids $b = \emptyset$) has expected type $\mathbb{E}(V|V \leq \tau)$ (formally, the bidder's type is distributed according to the truncation of F on the interval $[\underline{v}, \tau]$), (ii) upon observing bid b the receiver beliefs to face type $v = \beta^{-1}(b)$ (if such a type exists) and (iii) for all other bids the receiver believes the bidder's type is $\underline{v}$. Note that the expected payoff of type v who mimicks type v' is

$$G(v')v - m(v') + \Phi(v') = G(v')v - m^{\mathcal{M}}(v')$$

As $\beta^{\mathcal{M}}$ was an equilibrium of the auction without signaling, the above objective is maximized upon bidding $\beta(v)$, i.e., bidding as the true type. We have thus verified that the bidding strategy β together with the specified system of beliefs forms an equilibrium.

□

Proposition 3 implies revenue equivalence with the additional requirement that the auction is augmented by an optimal disclosure policy that reveals all submitted bids.

This finding has a couple of implications relevant in practice. First, without an optimal disclosure policy revenue is typically lower than Equation (11). An example where an optimal disclosure policy is used is takeover bidding, where regulations require all bidding to be public. Our findings imply, that the specific payment rule has no further impact on revenue. Second, with a given non-optimal disclosure policy revenue equivalence does not obtain. Take for instance a first- and a second-price auction that reveal the winner's payment. Inference on winning and losing bidders' types is different, hence the signaling values differ. As a consequence, when the choice of a disclosure policy is restricted, auction design is critical. For instance, in public procurement regulations require disclosure of the final price but do not allow any further revelation of bids. When bidders have non-linear signaling concerns it is then a non-trivial task to decide on the optimal auction format.

As in the case of linear signaling concerns, we can look for the optimal participation threshold $\tau^\star$ in the auction. The bidders' signaling concerns induce the auctioneer to reduce the threshold for participation below the optimal level without signaling.

Corollary 2. *Assume virtual valuations are increasing. In the revenue maximizing auction more bidders participate than if bidders had no signaling concern, i.e.* $\tau^\star < \tau^{\mathcal{M}}$*.*

Proof. It follows similar steps as the proof of Corollary 1. □

Concave Signaling Concerns. With concave signaling concerns the total signaling value is maximal upon disclosing the least amount of information, i.e., only the winner's identity. As the examples in Section 3 already show, such an information disclosure yields a high outside option $\mathcal{W}_{\tau,\emptyset}$ and is typically not revenue maximizing. The auctioneer faces a trad-off between increasing the total signaling value and decreasing the bidders' outside option. This trade-off is also affected by the induced participation threshold τ.

With full participation, i.e., $\tau = \underline{v}$, every bidder submits a bid in equilibrium. Hence, disclosing whether a bidder submitted a bid does not provide additional information to an outside observer. But it allows for reducing the bidders' outside option, because abstention will then be punished by the most pessimistic belief (that the mean is $\underline{v}$). Hence, disclosing only the winner's identity together with information on bidders who actually submitted a bid yields maximal signaling value and minimal outside, and therefore maximal revenue. The following Proposition shows that this continues to hold if the participation threshold is sufficiently low.

More precisely, consider first the signaling value, given by the term $\int \Phi(v)\mathrm{d}H_\tau^A(v)$. Information disclosure amounts to choosing a distribution over posterior means H_τ^A, where H_τ^A is a mean-preserving spread of the *minimal information* distribution $H_\tau^{\min}$. The latter distribution is given by the policy that discloses only the winner's identity, if the auction has a winner.

Before we can state the Proposition, let us define H_τ^P the distribution over posterior means that arises from a disclosure policy which reveals the winner's identity and whether a bidder participated in an auction when the participation threshold is $\tau \in [\underline{v}, \overline{v}]$. As discussed above, we have $H^P_{\underline{v}} = H^{\min}_{\underline{v}}$. Moreover, define

$$Rev^P(\tau) = Rev^{\mathcal{M}}(\tau) + n\Big(\int_{\underline{v}}^{\overline{v}} \Phi(v) \mathrm{d}H^P_\tau(v) - \mathbb{E}[V|V < \tau]\Big). \tag{13}$$

Proposition 4. *Consider a standard auction, in which a bidder participates whenever his type is above τ, and upon participation follows a strictly increasing bidding strategy.*

(i) If participation is fully observable we have that $Rev(\tau) \leq Rev^P(\tau)$.

(ii) There exists $\tau' > \underline{v}$ such that $Rev(\tau) \leq Rev^P(\tau)$ whenever $\tau < \tau'$.

Proof. From Proposition 1 we have that revenue equals (3). With observable participation we have $\mathcal{W}^A_{\tau,\emptyset} = \mathbb{E}[V|V < \tau]$, independent of the specific auction format. Furthermore, because Φ is concave the signaling value $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_\tau(v)$ is maximal when only the winner's identity is disclosed in addition, i.e., when $H^A_\tau = H^P_\tau$. This proves (i).

To prove (ii), note that for every auction in which participation is not fully observable we have $\mathcal{W}^A_{\tau,\emptyset} > \mathbb{E}[V|V < \tau]$. Moreover, for $\tau = \underline{v}$ we have $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_{\underline{v}}(v) \leq \int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^{\min}(v)$ by concavity of Φ. Hence, $Rev(\underline{v}) < Rev^P(\underline{v})$. Both $\mathcal{W}^A_{\tau,\emptyset}$ and $\int_{\underline{v}}^{\overline{v}} \Phi(v)\mathrm{d}H^A_\tau(v)$ are continuous in the participation threshold τ, hence (ii) follows by continuity from the previous assertion.

□

Proposition 4 derives an upper bound for the revenue if either participation is fully observable, or many bidder types participate. In other cases, i.e., when few bidder types participate in the auction, other information disclosure policies than revealing the winner's identity and whether a bidder participated leads to higher revenue. In order to increase the signaling value some participation decisions have to be concealed, which then increases the bidders' outside option.

Proposition 4 reveals a fundamental difference between pure information design and mechanism design with information disclosure. In information design the sender has costless access to all information, while in mechanism design the information is privately held by the agents. In our setting the auctioneer benefits from revealing additional information, namely whether a bidder participated. Such disclosure reduces the value of the bidders' outside option, i.e., the expected signaling value from non-participation. The reduced outside option allows the auctioneer to extract more revenue from bidders. The benefit from revealing the bidders' participation is larger, the lower the participation threshold τ. For $\tau \approx \underline{v}$ it becomes optimal to disclose only the winner's identity and all participation decisions.

The bound derived in Proposition 4 stems from a disclosure policy that reveals only the winner's identity, and a list of participating bidders. In particular, revealing additional information such as the winner's payment in first- or second-price auctions reduces revenue. In some contexts it is mandatory to disclose such payments. Does this necessarily imply that revenue strictly reduces? We show that in theory there is a remedy to eliminate the adverse effects of information leakage via observable payments.

Proposition 5. *For every $\varepsilon > 0$ and every τ there is an auction with mandatory disclosure of all payments that exhibits an equilibrium with strictly increasing bidding strategies, for which $Rev(\tau) > Rev^P(\tau) - \varepsilon$.*

Proof. See the appendix. □

Remark 1. With concave signaling concerns an auction does not necessarily constitute an optimal mechanism, hence our focus on equilibria in strictly increasing strategies is restrictive. To see this let us consider the following example with 2 bidders, valuations drawn from a uniform distribution on $[1, 2]$, and a signaling function $\Phi(v) = k\sqrt{v}$ with $k > 0$. Therefore, the optimal auction uses no reserve price, and participation is fully observable. Following Propositions 4 and 5, the maximal revenue of an auction is then

$$Rev^A = \frac{4}{3} + 2k\left(\frac{1}{2}\sqrt{\frac{5}{3}} + \frac{1}{2}\sqrt{\frac{4}{3}}\right).$$

Now consider a lottery, in which the object is allocated at random. Provided that both bidders participate, type v's expected utility is $\frac{1}{2}v + k\sqrt{\frac{1}{2}}$. Note that the allocation, i.e., who is assigned the object, does not reveal new information about the bidders' types. To ensure full participation the seller can charge a participation fee of $\frac{1}{2} + k\sqrt{\frac{3}{2}}$, thus revenue is

$$Rev^P = 1 + 2k\sqrt{\frac{3}{2}}.$$

Clearly, for sufficiently large k we have $Rev^P > Rev^A$.[19] Recall that we assume the final allocation (i.e., who gets the object) is observable. Therefore, a mechanism that implements an allocation rule that conditions on reported types necessarily reveals information about bidders. With a concave signaling function, information revealed via the allocation reduces the signaling value the auctioneer can extract. If signaling concerns are sufficiently strong, the gain in signaling value outweighs the loss in terms of standard revenue $Rev^{\mathcal{M}}$, hence an auction is no longer optimal. Whether a lottery represents an optimal mechanism depends on the marginal gain from improving the allocation versus the marginal loss in terms of signaling value this brings about.[20]

[19] Straightforward computations lead to a threshold of $\hat{k} \approx 87.84$. This threshold would reduce with either more bidders, more concave signaling function Φ, or a more spread-out distribution.

[20] However, note that a formal analysis is all but straightforward. The presence of signaling prevents us from using standard methods, such as pointwise maximization of the objective.

6 Discussion

In this paper, we analyze optimal auctions in an independent private values environment with signaling, i.e. where bidders care about the perception of a third party. To keep the analysis concise and tractable we focused on linear, convex and concave signaling concerns. The results of Dworczak and Martini (2019) indicate that the disclosure policy maximizing the signaling value for general preferences is a combination of intervals where the type is fully disclosed and intervals on which types are fully pooled. However, it is not straightforward to translate such a disclosure rule into a payment rule for a standard auction. Understanding the polar cases of convexity and concavity allows us to address a preference for the aftermarket that has been studied in the literature on information design, namely where Φ is a distribution function.[21] Under regularity conditions, there is a unique value $\hat{v}$ such that Φ is convex on $[\underline{v}, \hat{v}]$ and concave on $[\hat{v}, \overline{v}]$. Hence, if the participation threshold is sufficiently high we are back in the concave case. Otherwise, maximizing revenue calls for revealing low bids while at the same time pooling higher bids. Disclosure of low value implies that the auctioneer again prefers to disclose whether a bidder participated.

A natural follow-up question concerns the extent to which our results can be generalized to a richer class of mechanisms. Beyond mechanism design, that could provide new and exciting perspectives in applied fields such as advertising, marketing science and industrial organization. For instance, the literature on conspicuous consumption (e.g., Bagwell and Bernheim (1996) and Corneo and Jeanne (1997)) studies product markets where the consumption value depends on the belief of a social contact. A profit-maximizing seller will try to exploit this by tailoring its product line and prices to the information revealed by the consumer's choice. That will lead to new insights about consumer behavior and firm strategies that exploit signaling concerns.[22]

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[21] Rayo and Segal (2010) study such a sender–receiver game. The receiver chooses a binary action $a \in \{0, 1\}$. Choosing $a = 0$ yields a fixed utility r which is distributed according to some distribution function G. Choosing $a = 1$ yields utility θ, where θ is the sender's private information. The sender wants to maximize the probability of choosing $a = 1$. Hence, the sender's reduced form utility is $G(\mathbb{E}(\theta))$.

[22] See Rayo and Segal (2010) and Friedrichsen (2018) for analyses into that direction.

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A Proof of Proposition 5

Consider the following variant of a first-price auction: Bidders submit non-negative bids, the bidder submitting the highest bid wins and with exogenous probability $1 - \varepsilon$ makes no payment, but pays his own bid with probability ε.[23] Every bidder has to pay the entry fee φ before submitting a bid. The expected profit of a bidder of type v upon entering the auction and bidding as if he was type v' is

$$\Pi(v|v') = G(v')(v - \varepsilon\beta(v')) + \mathcal{W}_\tau(v') - \varphi.$$

[23] The superscript A is omitted in the proof, as it goes through a specific first-price auction.

From the first-order condition we get

$$\beta^\star(v) = \frac{1}{\varepsilon}\beta^{\mathcal{M}}(v) + \frac{\mathcal{W}_\tau(v) - \mathcal{W}_\tau(\tau)}{\varepsilon G(v)},$$

where $\beta^{\mathcal{M}}$ is the bidding strategy in a first-price auction without signaling and entry fee that induces only types above τ to participate. Note that we have used the fact that $\beta(\tau) = 0$, which is true because in equilibrium type τ only wins the auction when no other bidder enters and is thus not willing to bid a strictly positive amount. Furthermore, to induce participation for all types above τ the fee has to satisfy

$$\mathcal{W}_{\tau,0} = G(\tau)\tau + \mathcal{W}_\tau(\tau) - \varphi \quad \Leftrightarrow \quad \varphi = G(\tau)\tau + \mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}.$$

The revenue is thus given by

$$\begin{aligned} Rev^{\varepsilon}(\tau) &= n(1-F(\tau))\varphi + (1-F^n(\tau))\mathbb{E}\big[\varepsilon\beta^\star(V_1)|V_1 \geq \tau\big] \\ &= n(1-F(\tau))\big(G(\tau)\tau + \mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}\big) + \int_\tau^{\overline{v}} \left(\beta^{\mathcal{M}}(s) + \frac{\mathcal{W}_\tau(s) - \mathcal{W}_\tau(\tau)}{G(s)}\right)\mathrm{d}F^n(s) \\ &= Rev^{\mathcal{M}}(\tau) + n(1-F(\tau))(\mathcal{W}_\tau(\tau) - \mathcal{W}_{\tau,\emptyset}) + n\int_\tau^{\overline{v}} \big(\mathcal{W}_\tau(s) - \mathcal{W}_\tau(\tau)\big)f(s)\mathrm{d}s \\ &= Rev^{\mathcal{M}}(\tau) + n\left[F(\tau)\mathcal{W}_{\tau,\emptyset} + \int_\tau^{\overline{v}} \mathcal{W}_\tau(s)f(s)\mathrm{d}s - \mathcal{W}_{\tau,\emptyset}\right]. \end{aligned} \tag{14}$$

Note that

$$\begin{aligned} \mathcal{W}_\tau(s) &= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\cdot\Big(\varepsilon\Phi(s) + (1-\varepsilon)\Phi\big(v_{W,k}\big)\Big) \\ &\qquad + (1-F_k(s|\tau))\cdot\Big(\varepsilon\int_s^{\overline{v}}\Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) + (1-\varepsilon)\Phi\big(v_{L,k}\big)\Big)\Big\} \\ &= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\Phi\big(v_{W,k}\big) + (1-F_k(s|\tau))\,\Phi\big(v_{L,k}\big)\Big\} \\ &\quad -\varepsilon\sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1)\Big\{F_k(s|\tau)\Big(\Phi(s) - \Phi\big(v_{W,k}\big)\Big) \\ &\qquad +(1-F_k(s|\tau))\Big(\int_s^{\overline{v}}\Phi\big(v_{L,k}(x)\big)\mathrm{d}F_k(x|\tau) - \Phi\big(v_{L,k}\big)\Big)\Big\}, \end{aligned}$$

where $\mathcal{B}_{n-1,F(\tau)}(k-1) := \binom{n-1}{k-1}F(\tau)^{n-k}(1-F(\tau))^{k-1}$, $v_{W,k} := \mathbb{E}[V_1|V_k \geq \tau > V_{k+1}]$, $v_{L,k} := \mathbb{E}[V|V_1 > V \geq V_k \geq \tau > V_{k+1}]$, $v_{L,k}(s) := \mathbb{E}[V|V_1 = s, s > V \geq V_k \geq \tau > V_{k+1}]$ and $F_k(v|\tau) := \big(\frac{F(v)-F(\tau)}{1-F(\tau)}\big)^{k-1}$ for all $k = 1,\ldots,n$ denotes the conditional probability of the maximum of the $k-1$ other bids if all of these exceed τ. Hence,

$$\begin{aligned}
\int_{\tau}^{\overline{v}} \mathcal{W}_{\tau}(s) f(s) \mathrm{d}s &= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1) \Big\{ \tfrac{1}{k} \Phi\big(v_{W,k}\big) + \tfrac{k-1}{k} \Phi\big(v_{L,k}\big) \Big\} \\
&\quad - \varepsilon \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1) \Big\{ \int_{\tau}^{\overline{v}} F_k(s|\tau) \Big(\Phi(s) - \Phi\big(v_{W,k}\big) \Big) f(s) \mathrm{d}s \\
&\quad + \int_{\tau}^{\overline{v}} (1 - F_k(s|\tau)) \Big(\int_{s}^{\overline{v}} \Phi\big(v_{L,k}(x)\big) \mathrm{d}F_k(x|\tau) - \Phi\big(v_{L,k}\big) \Big) f(s) \mathrm{d}s \Big\} \\
&= \sum_{k=1}^{n} \mathcal{B}_{n-1,F(\tau)}(k-1) \Big\{ \tfrac{1}{k} \Phi\big(v_{W,k}\big) + \tfrac{k-1}{k} \Phi\big(v_{L,k}\big) \Big\} - \varepsilon C,
\end{aligned}$$

where by concavity of Φ and compactness of the support of the bidders' valuations we have $C > 0$ and finite. Plugging the above expression back into (14) and noting that $\mathcal{W}_{\tau,\emptyset} = \mathbb{E}[V|V < \tau]$ (because participation is observable) yields $Rev^{\varepsilon}(\tau) = Rev^{P}(\tau) - \varepsilon C \to Rev^{P}(\tau)$ as $\varepsilon \to 0$.